Optimal Fiscal and Monetary Policy with Collateral Constraints

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Abstract

We study the Ramsey optimal fiscal and monetary policy in an economy where banks face collateral constraints. Inflation reduces the net worth of banks and tightens their collateral constraint by revaluing their nominal assets and liabilities. The optimal policy balances tax distortions with the costs of inflation on banks, thereby deviating from perfect tax smoothing. Our quantitative analysis reveals that inflation plays a much smaller role in financing fiscal needs in the optimal policy compared to existing literature. When considering price stickiness and long-term government debt, optimal inflation is modest and persistent, and the role of inflation in fiscal financing increases with the maturity of government debt.

Key words: Collateral constraints; Optimal fiscal and monetary policy; Bank net worth

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1 Introduction

How should fiscal and monetary policy be determined in response to government spending shocks? Without defaulting on outstanding debt, a government can choose to either increase distortionary taxation or resort to inflation in order to reduce the real value of government debt denominated in domestic currency. Given the escalation of deficits and public debt to historical highs in major economies following the Great Financial Crises and the COVID-19 pandemic, many economists and commentators have argued in favor of a higher inflation as a means to alleviate the burden of public debt.[1]

While inflation can effectively alleviate the real burden of debt ex post, it is important to consider the associated costs. This paper introduces a novel cost of inflation on banks. Higher inflation reduces the net worth of banks by devaluing their nominal fixed-income claims. First, banks with balance sheet exposure to government debt directly face losses when the government monetizes its debt.[2] Second, due to the maturity mismatch between bank assets and liabilities, a persistent rise in the inflation rate (e.g., a higher inflation target) leads to a faster decline in the value of bank assets compared to liabilities. Banks play an important role in financial intermediation, and losses incurred by financially constrained banks impede credit supply and dampen real economic activity. A vivid example is the recent failure of Silicon Valley Bank (SVB) in March 2023, marking the second-largest bank failure in U.S. history. The immediate cause of the failure was a run on SVB, triggered by the announcement of significant losses resulting from the sale of its mortgage-backed securities. These losses were prompted by an increase in interest rates, as a response by the Federal Reserve to counter rising inflation.

In this paper, we show that incorporating the cost of inflation on banks significantly alters the prescription of optimal fiscal and monetary policy. Specifically, we find a substantially weaker role of inflation as a means to finance fiscal needs when compared to standard models. Thus, our analysis highlights a novel rationale for the undesirability of government debt monetization,

[1] For example, see Rogoff (2008), Blanchard et al. (2010), and Kelton and Chancellor (2020). Of course, the desirability of higher inflation extends beyond reducing the public debt burden. It is also advocated for addressing wage rigidity, reducing household debt, and mitigating the impact of the zero lower bound.

[2] For instance, the holdings of Japanese government bonds by Japan's banks amounted to 900% of their Tier 1 capital in 2012 (Jenkins and Nakamoto, 2012). When the Bank of Japan started qualitative and quantitative monetary easing in 2013 with the goal of reaching the 2% inflation target, fears arose that banks would bear large losses if the inflation rate were raised.

emphasizing the interaction between financial frictions and bank balance sheet costs of inflation.

In our baseline model economy, we consider a flexible-price environment. The economy is populated with a large number of bankers who provide funds to non-financial firms. Firms are exposed to idiosyncratic productivity shocks. When a high-productivity firm seeks to expand its productive resources, its banker must raise external funds through collateralized borrowing. Bankers hold nominal government debt and physical capital, which serve as collateral when they borrow from other bankers. The government finances its fiscal expenditures and interest payments by imposing distortionary labor taxes and using state-contingent inflation.

We use the model to study the response of Ramsey optimal fiscal and monetary policy to fiscal expenditure shocks. When the government generates inflation to reduce the real value of debt, bankers' collateral constraints are tightened, which impedes resource reallocation across firms with different productivities. Additionally, as firms anticipate future tightening of collateral constraints due to a reduction in the real value of government bonds, they also decrease their investment in physical capital. The optimal policy response to these shocks entails a combination of higher tax rates and higher inflation rates, as the government seeks to balance the costs associated with distortionary taxes and inflation. This stands in contrast with standard models without financial frictions, which recommend the use of state-contingent inflation to smooth tax distortions across time and states (Lucas and Stokey, 1983; Chari et al., 1991). In these models, inflation is effectively a lump-sum tax on government debt holders from an ex post perspective.

We calibrate the model to the postwar U.S. economy. To quantify how the Ramsey optimal policy finances increases the government spending, we conduct a fiscal financing decomposition. This decomposition analyzes the fractions of the present value of expected increases in government spending that are financed through increases in inflation, higher taxes, and decreases in future real interest rates. In the optimal policy of our model economy, 56% of the increase in fiscal needs is financed through higher inflation, while 52% is financed through higher tax revenues.[3] In contrast, standard models without the collateral constraints of bankers rely solely on higher inflation to finance all increases in government expenditures, with tax revenues remaining constant.[4] This tradeoff between the cost of inflation on banks and tax distortions is further illustrated by

[3] Higher future real interest rates have a negative contribution of -7% on fiscal financing.

[4] In Appendix B, we study how the fiscal financing decomposition results are affected by variations in different model parameters.

comparing the Ramsey optimal policy with two alternative policies in the baseline model. One alternative policy involves maintaining a constant inflation rate regardless of fiscal shocks, while the other alternative policy entails maintaining a constant tax rate instead.

Price stickiness is another reason levied against the use of inflation in the optimal policy (Schmitt-Grohé and Uribe, 2004; Siu, 2004). However, when the government issues long-term debt, large changes in the real value of its debt can be produced by modest and persistent inflation. Therefore, inflation still plays an important role in fiscal financing in the presence of nominal rigidity (Leeper and Zhou, 2021; Sims, 2013). Accordingly, we extend our baseline model to incorporate price stickiness. With the degree of price stickiness and the maturity of debt calibrated to the U.S. economy, we find that it is optimal for the government to finance 31% of the increased fiscal spending through higher inflation. Consistent with the data, the optimal response of inflation is modest and persistent. Specifically, 26% of the increase in fiscal spending is financed by higher inflation in future periods, whereas only 5% is financed by higher inflation in the initial period when the fiscal shock occurs. The maturity of government debt significantly influences policy recommendations. For instance, when the debt matures in one period (quarter), higher inflation only finances 14% of higher government spending in the optimal policy. This finding is consistent with previous studies, indicating that long-term government debt can help alleviate the costs of inflation resulting from nominal rigidity. In contrast, long-term government debt is unable to alleviate the costs of inflation on banks.

Finally, we explore two applications of the model. In the first application, we analyze the implications of the model for wartime financing in the conflicts in Iraq, Afghanistan, and Syria. The budgetary costs of these wars exceeded 7% of total government consumption at their peak. In our comprehensive model with bank balance sheet costs of inflation, price stickiness, and long-term debt, the optimal policy response to these wartime expenditures results in an average increase in the annual inflation rate of 0.50%, accompanied by an average rise in the labor tax rate of 1.31 percentage points. In comparison, in a standard model without collateral constraints or price stickiness, the optimal policy entails an average increase in the annual inflation rate of nearly two percent, with minimal adjustments to the tax rate. In the second application, we study the financing of the sharp increase in government expenditures during the COVID-19 pandemic. In the model with bank balance sheet costs of inflation, price stickiness, and long-term debt, the average increase in the inflation rate from 2020 to 2023 is 1.90%, whereas in a standard model, it is 3.28%.

Related Literature. Our paper is related to an extensive theoretical and empirical literature on the liquidity role of government bonds or fiat money in the presence of financial frictions (e.g., Woodford, 1990; Holmstrom and Tirole, 1998; Aiyagari and McGrattan, 1998; Krishnamurthy, 2002; Kiyotaki and Moore, 2019; Azzimonti and Yared, 2019; Bassetto and Cui, 2021). In particular, we build on the work of Angeletos et al. (2013), who study optimal fiscal policy when real government debt serves as private collateral and focus on the determination of the long-run debt level. In this paper, we introduce nominal government debt and emphasize the importance of collateral constraints in shaping monetary policy. We argue that inflation reduces the real value of government debt and impairs its effectiveness as a source of liquidity. In addition, we also explore the implications of nominal rigidity and long-term government debt. Martin (2013) investigates time-consistent optimal fiscal and monetary policy in a New Monetarist model, where the provision of public goods is funded through money, nominal bonds, and distortionary taxes. Martin (2012) applies a similar model to study the optimal financing of war expenditures. In contrast to these models, where money serves as the medium of exchange and government bonds have no liquidity value, our model has no money and government debt provides liquidity value. Furthermore, we study optimal policy with commitment.

This paper also contributes to the literature that examines the redistribution effect of inflation by revaluing nominal contracts in general equilibrium models. This literature shares the view that nominal contracts create a connection between inflation and the real economy, playing a significant role in monetary non-neutrality even under fully flexible prices. Several studies have investigated the redistribution effect of inflation on nominal household debt (Auclert, 2019; Garriga et al., 2017; Meh et al., 2010). Gomes et al. (2016) model the effect of unanticipated inflation on the real value of nominal corporate debt and the severity of debt overhang. Our work contributes to this literature by highlighting the importance of nominal positions of the banking sector. Corhay and Tong (2021) build a model examining the redistribution effect between financial intermediaries and the corporate sector through long-term nominal corporate debt and study optimal monetary policy rules. In comparison, our paper abstracts away from borrowing constraints faced by the corporate sector and focuses on the redistribution effect of inflation between banks and the government through government debts. Our study centers on exploring the implications of Ramsey optimal fiscal and monetary policy in this context.

At a conceptual level, this paper also relates to a growing literature that explores the relationship between sovereign default and bank fragility (Gennaioli et al., 2014; Bocola, 2016; Sosa-Padilla, 2018; Bolton and Jeanne, 2011; Guerrieri et al., 2013). A notable observation in this literature is the significant exposure of banking sectors in many countries to government debt, meaning that a government default directly impacts the value of banking sector assets. In our model, where inflation can be viewed as a partial default on government liabilities, we share the notion with this literature that the repudiation of government debt tightens financial constraints of the banking sector. This literature usually assumes a lack of commitment on the part of the government. In contrast, our analysis centers on optimal policy under full government commitment and places exclusive focus on the role played by financial frictions.

Empirical Relevance. How large is the effect of inflation on the real value of the assets, liabilities, and net worth of U.S. banks? In Cao (2019), we quantify this effect using bank-level data from the Bank Reports of Conditions and Income (call reports) filed quarterly by these banks. We first document that the average maturity of nominal assets is longer than nominal liabilities by about five years between 1997 and 2009, indicating a sizable maturity mismatch. To quantify the impact of inflation, following the approach of Doepke and Schneider (2006), we then study the effect of a hypothetical scenario of a 1% unanticipated and permanent increase in the inflation rate, leading to a parallel upward shift in the yield curve. Our results show that, on average, U.S. banks would experience a 15% loss of Tier 1 capital in this inflation scenario. Importantly, the size of the loss is similar for banks that do not hold interest rate derivatives and thus do not hedge this risk.[5] These results underscore the substantial impact of inflation on the real value of bank balance sheets. It suggests that even a moderate episode of inflation can have significant consequences for bank net worth.[6]

While Cao (2019) examines a hypothetical unanticipated increase in inflation, Corhay and Tong

[5]Many banks do not hold any interest rate derivatives. For instance, in 2009, only 40% of banks held interest rate derivatives. Recent empirical studies (Begenau et al., 2020; Gomez et al., 2021) indicate that banks do not fully hedge their exposure to interest rate risk and inflation risk. In particular, Begenau et al. (2020) demonstrate that banks incur similar exposures to interest rate risk through derivatives and other business activities. For both types of positions, increases in interest rates are detrimental.

[6]Our results align with a similar analysis conducted on Japanese banks (Bank of Japan, 2013). They find that a 1% parallel upward shift in the yield curve results in an average 20% loss of Tier 1 capital for Japanese banks in 2012.

(2021) investigate the impact of actual inflation surprises on financial intermediaries' stock returns. They find that a surprise increase in inflation expectations lowers stock returns for banks. Moreover, the magnitude of the drop is greater for banks with a larger maturity mismatch. Their result once again confirms that inflation has a negative impact on bank balance sheets.

Roadmap. The rest of the paper is organized as follows. We describe the benchmark model and the Ramsey optimal policy problem in Section 2 and explore the quantitative results in Section 3. In Section 4, we extend the model to incorporate price stickiness. Section 5 applies the model to study war financing. Finally, we conclude in Section 6.

2 Model

2.1 Environment

The economy consists of a continuum of identical households. Within each household reside equal masses of bankers $i \in [0, 1]$ and workers $j \in [0, 1]$. Members in each household share consumption perfectly. Each worker supplies labor in a competitive labor market and earns a wage income. Each banker channels funds to a firm that produces final goods. We ignore financial frictions between a banker and their firm; thus, each banker effectively owns the firm.[7] We use i to index the firm owned by banker i.

Preference and Technology. Preferences over stochastic processes for the household consumption $\{c_t\}_{t\geq 0}$ and labor supply $\{h_{j,t}\}_{\geq 0}$ of each worker j are ordered by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho} - 1}{1-\rho} - \chi \frac{\int_0^1 h_{j,t}^{1+\epsilon} dj}{1+\epsilon} \right). \tag{1}$$

Firm i uses $k_{i,t}$ units of physical capital and $n_{i,t}$ units of labor to produce output $y_{i,t}$:

$$y_{i,t} = z_{i,t} F(k_{i,t}, n_{i,t}),$$

[7]As each banker is the owner of a firm, in the text below, we use banker i and firm i interchangeably, with a slight abuse of notation. The assumption of no friction between bankers and firms is similar to Gertler and Kiyotaki (2010). It allows us to focus on the bankers' balance sheets and how their borrowing capacity is limited by their net worth. As in Gertler and Kiyotaki (2010), our analysis focuses on the financial constraints of banks (lenders) rather than borrowers (e.g., Bernanke et al., 1999).

where F has decreasing returns to scale, with $F(k,n) = k^\alpha n^\theta$ and $\alpha + \theta < 1$. $z_{i,t}$ is an idiosyncratic productivity shock independent and identically distributed across both bankers and time. $z_{i,t}$ can take two values:

$$z_{i,t} = \begin{cases} z^H & \text{with probability } \sigma \\ z^L & \text{with probability } 1-\sigma \end{cases}.$$

The idiosyncratic productivity shock generates a need for capital reallocation.

Physical capital depreciates at rate δ. Aggregate capital stock a_t is the sum of the stock of undepreciated capital and current investment i_t:

$$a_t = (1-\delta)a_{t-1} + i_t.$$

Aggregate Uncertainty. The only source of aggregate uncertainty in this model is a stochastic government consumption g_t. Aggregate history up until time t is $g^t = (g_0, ..., g_t)$, and the time-0 probability of g^t is denoted by $\Pr(g^t)$. To simplify the notation, we denote a random variable dependent on the history g^t as X_t.

The aggregate output y_t is allocated among household consumption, investment expenditures, and government consumption according to the social resource constraint, given by:

$$c_t + a_t + g_t = (1-\delta)a_{t-1} + y_t. \tag{2}$$

Government Policy. Within the government, there are fiscal and monetary authorities. The fiscal authority imposes proportional taxes on labor income, with a tax rate denoted as τ_t, and also issues nominal bonds represented by B_t. We model government bonds as securities that provide an infinite stream of nominal coupons, which decrease at a constant rate $\eta \in [0,1]$. Specifically, a bond issued in period t promises to pay $(1-\eta)^{s-1}$ dollars in period $t+s$, where $s \geq 1$ (Leeper and Zhou, 2021). The exogenous parameter η determines the average maturity of bonds. When $\eta = 1$, it indicates a one-period bond.

The monetary authority determines the nominal bond price Q_t^B paid on B_t. The following consolidated government budget constraint must hold:

$$\tau_t w_t h_t + \frac{Q_t^B B_t}{P_t} = \frac{1 + (1-\eta) Q_t^B}{P_t} B_{t-1} + g_t. \tag{3}$$

2.2 Capital market and collateral constraint

We now describe the collateral constraint in the capital market, which is the key component of the model. In Figure 1, we illustrate the sequence of activities within each time period t. At the beginning of period t, workers and bankers separate, and they cannot interact until the end of the period. Prior to the separation, each household distributes its accumulated assets from the previous period evenly among all the bankers within the household. Consequently, each banker i holds an equal share of the household's assets, which comprise physical capital a_{t-1} and government bonds B_{t-1}.[8]

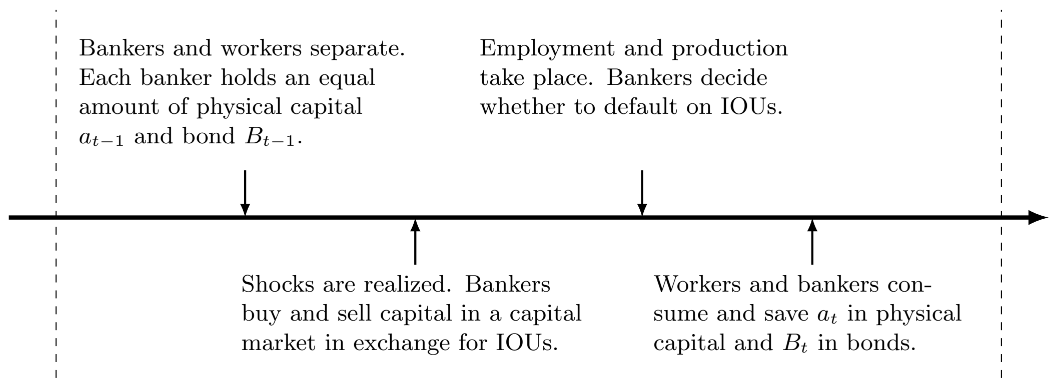

Figure 1: Timeline of activities within period t.

Following the realization of idiosyncratic and aggregate shocks, high-productivity bankers want to expand their production and require more capital than they currently possess ($k_{i,t} > a_{t-1}$ if $z_{i,t} = z^H$). In a competitive capital market, these bankers can purchase the additional capital amount $k_{i,t} - a_{t-1}$ from other bankers. The price of capital in this market is denoted as q_t units of consumption goods. Buyers of capital do not make the payment for the capital until production is finished; therefore, at this stage, they issue private IOUs to the sellers.

After employment and production take place, there is a possibility that buyers may default on their IOUs. In such cases, sellers could confiscate a portion of a buyer i's assets, which is ξ

[8]The assumption that bankers within the same household redistribute assets among themselves allows us to study heterogeneity and capital reallocation while maintaining the structure of a representative household.

fraction of capital installed in the firm $k_{i,t}$ and the total real payoff from government debt holding $\frac{1+(1-\eta)Q_t^B}{P_t}B_{t-1}$.[9] Therefore, buyers face an incentive constraint that limits the total value of IOUs they can issue to be less than or equal to the total value of confiscable assets, given by:

$$q_t\left(k_{i,t}-a_{t-1}\right) \leq \xi k_{i,t} + \frac{1+(1-\eta)Q_t^B}{P_t}B_{t-1}.$$

By rearranging the inequality constraint, we can express it as follows:

$$k_{i,t} \leq \underbrace{\frac{1}{q_t-\xi}}_{\text{leverage}} \times \underbrace{\left[q_t a_{t-1} + \frac{1+(1-\eta)Q_t^B}{P_t}B_{t-1}\right]}_{\text{net worth}}. \tag{4}$$

This constraint implies that level of capital $k_{i,t}$ is constrained by the banker's total net worth. The term $\frac{1}{q_t-\xi}$ represents the leverage, which signifies that for each unit of capital used in production, banker i could credibly pledge a fraction of ξ. As a result, the remaining fraction of $q_t-\xi$ needs to be secured using the banker's own net worth.

The model broadly captures the mismatch of maturity observed in bank balance sheets, as well as the negative effects of inflation on bank net worth. In this model, a portion of bankers' assets consists of nominal government bonds that mature in one period (when $\eta = 1$) or more periods (when $\eta > 1$). Therefore, inflation (e.g., a higher period-t price level P_t) reduces the real value of bank assets. On the other hand, bankers' liabilities are only within-period, and their real value remains unaffected by inflation. As a result, inflation reduces the net worth of bankers and tightens their collateral constraints.

2.3 Households' decision problem and competitive equilibrium

The production decisions vary across firms solely based on the current productivity shock since all bankers have identical asset holdings before shocks realize. Consequently, variables related to production decisions are denoted with a superscript s, where $s = L$ when $z_{i,t} = z^L$, and $s = H$

[9] Due to spatial separation, bankers and workers cannot reshuffle the resources among themselves after shocks realize. For the same reason, a banker cannot pledge the wage incomes of workers in the same household as collateral. A banker cannot credibly pledge his or her own future income either. This assumption that human capital is inalienable has been followed in much of the literature on financial frictions since Hart and Moore (1994). As a buyer's default happens after production when physical capital can be converted to consumption goods one for one, the real price of capital $k_{i,t}$ at this point is one.

when $z_{i,t} = z^H$. As workers are homogeneous, they work the same amount, and $h_{j,t} = h_t$ for all j.

Since all household members share their consumption risks, a household faces a consolidated end-of-period budget constraint:

$$c_t + a_t + \frac{Q_t^B B_t}{P_t} = (1 - \tau_t)\, w_t h_t + q_t a_{t-1} + \frac{1 + (1-\eta) Q_t^B}{P_t} B_{t-1} + \left[\sigma v_t^H + (1-\sigma) v_t^L\right]. \quad (5)$$

In this equation, a household's income consists of after-tax labor income earned by workers, income from household savings and profits v_t^s generated by a banker of type s from their firm, given by:

$$v_t^s = z^s F\left(k_t^s, n_t^s\right) - w_t n_t^s - \left[q_t - (1-\delta)\right] k_t^s. \quad (6)$$

A household's decision problem is involves optimizing the choices of $\{k_t^s, n_t^s, h_t, c_t, a_t, B_t\}_{t \geq 0}$ to maximize utility (1), subject to the end-of period budget constraint (5) and the collateral constraint (4). The workers' labor supply decision satisfies:

$$(1 - \tau_t)\, w_t = \chi \frac{h_t^\epsilon}{c_t^{-\rho}}. \quad (7)$$

The labor and capital demand conditions of a type-s bank are given by:

$$z^s F_n\left(k_t^s, n_t^s\right) = w_t, \quad (8)$$

$$z^s F_k\left(k_t^s, n_t^s\right) = q_t - (1-\delta) + \mu_t^s, \quad (9)$$

where $\mu_t^s U_{c,t}$ is the multiplier on the collateral constraint.

The market clearing conditions for the labor market and capital market are:

$$\sigma n_t^H + (1-\sigma) n_t^L = h_t,$$

$$\sigma k_t^H + (1-\sigma) k_t^L = a_{t-1}.$$

The aggregate economy. As detailed in Appendix A.1, we show that aggregate output and key prices can be expressed as functions of aggregate capital a_t, labor h_t, and the allocation of capital between the two types of bankers denoted by $x_t \equiv \frac{k_t^H}{a_{t-1}}$. Aggregate output satisfies

$$y_t = \Gamma(x_t) a_{t-1}^{\alpha} h_t^{\theta}, \tag{10}$$

where

$$\Gamma(x) = \left[\sigma z^{H\frac{1}{1-\theta}} x^{\frac{\alpha}{1-\theta}} + (1-\sigma) z^{L\frac{1}{1-\theta}} \left(\frac{1-\sigma x}{1-\sigma}\right)^{\frac{\alpha}{1-\theta}}\right]^{1-\theta}.$$

The endogenous total factor productivity (TFP) $\Gamma(x)$ achieves its maximum at an interior point x^*, given by

$$x^* \equiv \arg\max_x \Gamma(x) = \frac{z^{H\frac{1}{1-\alpha-\theta}}}{\sigma z^{H\frac{1}{1-\alpha-\theta}} + (1-\sigma) z^{L\frac{1}{1-\alpha-\theta}}}.$$

Intuitively, due to the production technology's decreasing returns to scale, there is an optimal level of capital allocation, denoted as x^*, in the absence of collateral constraints. However, when the constraint of high-productivity bankers becomes binding, the capital allocations are suboptimal, resulting in $x_t < x^*$. As a consequence, the TFP falls below the efficient level represented by $\Gamma(x^*)$.

In the equilibrium, the marginal product of labor is equalized between the two types of bankers since there are no frictions in the labor market. Consequently, the real wage rate is equal to the aggregate marginal product of labor:

$$w_t = \theta \Gamma(x_t) a_{t-1}^{\alpha} h_t^{\theta-1}. \tag{11}$$

The price of capital is expressed as:

$$q_t = 1 - \delta + \underbrace{\alpha \Gamma(x_t) a_{t-1}^{\alpha-1} h_t^{\theta}}_{\text{aggregate MPK}} \underbrace{\left[z^L \left(\frac{1-\sigma x_t}{1-\sigma}\right)^{\alpha+\theta-1} \Gamma(x_t)^{-1}\right]^{\frac{1}{1-\theta}}}_{\text{deviation from aggregate MPK}} \equiv \mathbf{q}(a_{t-1}, h_t, x_t). \tag{12}$$

The term in the square brackets equals one if and only if $x_t = x^*$. Otherwise, it is less than one. In the equilibrium, the collateral constraint never binds for low-productivity bankers, as they sell capital ($k_t^L < a_{t-1}$). Therefore, $\mu_t^L = 0$ always holds. However, when the collateral constraint strictly binds for high-productivity bankers ($x_t < x^*$), the price of capital q_t deviates from what would be implied by the aggregate marginal product of capital. The multiplier on the collateral constraint for high-productivity bankers is given by:

$$\mu_t^H = \frac{1}{\sigma}\Gamma'\left(x_t\right) a_{t-1}^{\alpha-1} h_t^{\theta} \equiv \boldsymbol{\mu}^{\mathbf{H}}(a_{t-1}, h_t, x_t). \tag{13}$$

Here, μ_t^H is zero if and only if $x_t = x^*$ (and therefore $\Gamma'\left(x_t\right) = 0$).

The presence of the collateral constraint also introduces a wedge on the inter-temporal margin, as reflected in the Euler equations of the household:

$$U_{c,t} = \beta \mathbb{E}_t U_{c,t+1} q_{t+1} \left(1 + \frac{\sigma \mu_{t+1}^H}{q_{t+1} - \xi}\right), \tag{14}$$

$$U_{c,t} = \beta \mathbb{E}_t U_{c,t+1} \frac{1 + (1-\eta) Q_{t+1}^B}{Q_t^B \pi_{t+1}} \left(1 + \frac{\sigma \mu_{t+1}^H}{q_{t+1} - \xi}\right), \tag{15}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ represents the gross inflation rate. If the collateral constraint of high-productivity bankers strictly binds with a positive probability in period $t+1$, the corresponding Lagrange multiplier introduces a wedge between the rate of return of capital (or government bonds) and the intertemporal marginal rate of substitution. This wedge distorts the household's investment decision. Additionally, government bonds are priced at a premium relative to an asset that is an equally good form of saving but cannot be used as collateral.[10] This premium reduces the debt serving costs and allows the government to reduce taxes.

A *competitive equilibrium* is defined as follows. The household (workers and bankers) solves their optimization problems, taking prices as given. The wage rate, capital price, and bond price clear the labor, capital, and bond markets, respectively. In addition, the government budget constraint is satisfied. The equilibrium can be summarized by a set of allocations $\{y_t, a_t, h_t, c_t, x_t, B_t\}_{t\geq 0}$ and prices $\{q_t, w_t, P_t, \mu_t^H\}_{t\geq 0}$ that satisfy equations (3)–(5), (7), (10)–(15), $\mu_t^H \geq 0$, and the complementary slackness condition, given fiscal and monetary policies $\{\tau_t, Q_t^B\}_{t\geq 0}$, initial household asset positions a_{-1} and B_{-1} and the process of government consumption shocks $\{g_t\}_{t\geq 0}$.

[10]This is consistent with the observations that government bonds pay a lower return due to their liquidity attributes (Krishnamurthy and Vissing-Jorgensen, 2012) and that the "natural rate of interest" declines as credit tightens (Eggertsson and Krugman, 2012).

2.4 Ramsey optimal policy

The Ramsey optimal fiscal and monetary policy is the process $\{\tau_t, Q_t^B\}_{t\geq 0}$ associated with the competitive equilibrium that yields the highest social welfare:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho}}{1-\rho} - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right) \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, h_t\right).$$

We adopt the primal approach commonly used in the Ramsey policy literature. This approach involves substituting prices and policy instruments, allowing the Ramsey planner to directly determine real allocations. In equations (11)-(13), we expressed $\{w_t, q_t, \mu_t^H\}$ as functions of $\{a_{t-1}, h_t, x_t\}$. The labor tax rate is the wedge between the marginal rate of substitution and the marginal product of labor:

$$\tau_t = 1 + \frac{U_{h,t}}{U_{c,t}} \frac{1}{\Gamma(x_t) F_h(a_{t-1}, h_t)}. \tag{16}$$

Define the real value of government bond as $b_t = \frac{Q_t^B B_t}{P_t}$ and the real holding-period return on government bond as $r_t^b = \frac{1+(1-\eta)Q_t^B}{Q_{t-1}^B \pi_t}$. By rearranging and substituting the household budget constraint (5), the Euler equations (14)-(15), and equation (16), we can derive the flow implementability constraint

$$\beta \mathbb{E}_{t-1}\left[U_{c,t} c_t + U_{h,t} h_t - U_{c,t}(1-\alpha-\theta) y_t\right] + \beta \mathbb{E}_{t-1} U_{c,t}\left(a_t + b_t\right) = U_{c,t-1}(a_{t-1} + b_{t-1}). \tag{17}$$

Furthermore, we can combine the collateral constraint (4) with the government budget constraint (3) to substitute for r_t^b. This leads to the following expression:

$$x_t a_{t-1}\left(q_t - \xi\right) \leq q_t a_{t-1} + \left(\theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t\right). \tag{18}$$

We can establish the equivalence between the primal approach and the original Ramsey problem. An aggregate allocation $\{a_t, h_t, x_t, c_t, b_t\}_{t\geq 0}$ can be supported as a competitive equilibrium under appropriately chosen $\{r_t^b, \tau_t\}_{t\geq 0}$ if and only if it satisfies the social resource constraint (2), the flow implementability constraint (17), the Euler equation (14), the collateral constraint (18), $\boldsymbol{\mu}^{\mathbf{H}}(.) \geq 0$, and the complementary slackness condition. Price functions $\boldsymbol{q}(.)$ and $\boldsymbol{\mu}^{\mathbf{H}}(.)$ are defined in equations (12) and (13). The proof is detailed in Appendix A.2.

In the primal approach, the determination of the real state-contingent return on debt, denoted as r_t^b, can be achieved through state-contingent inflation. For example, when the government debt has a maturity of one period ($\eta = 1$), the state-contingent return $r_t^b = \frac{1}{Q_{t-1}^B \pi_t}$ can be adjusted in date t by choosing the inflation rate π_t. In the case of government debt with a longer maturity ($\eta < 1$), the adjustment in the real return $r_t^b = \frac{1+(1-\eta)Q_t^B}{Q_{t-1}^B \pi_t}$ can be achieved through changes in both the bond price Q_t^B and the inflation rate π_t. The bond price in turn depends on expected inflation in future periods, as indicated by the Euler equation (15). Note that the steady-state inflation rate is not determined in the Ramsey problem. Intuitively, steady-state inflation is factored into the pricing of nominal bonds, which leaves all real allocations and the real return on bonds unchanged. Without loss of generality, we assume zero steady-state inflation.[11]

In the remainder of the paper, we compare the Ramsey optimal policy in our model with that in an otherwise identical model but without the bank collateral constraint. In this comparison economy, the allocation of capital is always optimal (i.e., $x_t = x^*$). We also adopt a primal approach in this alternative economy. Specifically, allocations $\{a_t, h_t, c_t, b_t\}_{t\geq 0}$ can be supported as a competitive equilibrium in an economy without collateral constraints under given $\{r_t^b, \tau_t\}_{t\geq 0}$ if and only if the social resource constraint (2), the flow implementability constraint (17), and the Euler equation (14) hold with $x_t = x^*$. More details are provided in Appendix A.3.

The time-0 Ramsey problem differs from that of $t \geq 1$ due to the lack of previous commitment from the government. In the remainder of the paper, we study how the Ramsey policy responds to government consumption shocks around the non-stochastic steady state of the Ramsey economy. This can be viewed as an example of optimal policy under commitment from a timeless perspective (Woodford, 2003). By abstracting away from the government commitment issue, our sole focus is

[11] For example, when the government debt has a maturity of one period ($\eta = 1$), the Ramsey policy maker achieves the desired state-contingent return on bonds by choosing the state-contingent component of inflation:

$$\frac{\mathbb{E}_{t-1} r_t^b}{r_t^b} = \frac{\mathbb{E}_{t-1} \frac{1}{Q_{t-1}^B \pi_t}}{\frac{1}{Q_{t-1}^B \pi_t}} = \pi_t \mathbb{E}_{t-1} \frac{1}{\pi_t}.$$

The expected (inverse) inflation $\mathbb{E}_{t-1} \frac{1}{\pi_t} = 1$ is not determined in the optimal policy. When the government debt has a long maturity ($\eta < 1$), expected future inflation affects the bond price Q_t^B and thus the real return r_t^b. However, the steady-state inflation rate is still not determined. The result of zero steady-state inflation can emerge from a sticky-price version of our model (see Section 4) with a tiny amount of price stickiness. In the literature, steady-state inflation can be determined either by incorporating price stickiness, which drives the steady-state inflation rate to zero, or by introducing a non-interest-bearing government liability (money stock) that leads to the Friedman rule. Both features are absent in our flexible-price model.

on examining the influence of the collateral constraint on the design of the optimal policy. This approach allows us to establish a clean benchmark.

2.5 Fiscal financing decomposition

We are particularly interested in examining the source of fiscal financing in the Ramsey optimal policy following a government consumption shock and how the presence of collateral constraints affects this source of financing. To achieve this, we conduct the following decomposition.

Consider a scenario where an economy, with or without collateral constraints, has remained at the Ramsey-policy steady state until a government consumption shock occurs at time t. There are no further shocks from date $t+1$ on. We begin with the government budget constraint:

$$T_t + b_t = r_t^b b_{t-1} + g_t,$$

where $T_t = \tau_t w_t h_t$ is the labor tax revenue. By linearizing this equation, we obtain

$$\tilde{b}_{t-1} = \frac{1}{\bar{r}^b}\tilde{b}_t - \frac{1}{\bar{r}^b}\hat{r}_t^b - \frac{\bar{g}}{\bar{r}^b\bar{b}}\tilde{g}_t + \frac{\bar{T}}{\bar{r}^b\bar{b}}\tilde{T}_t,$$

where $\bar{X}$, $\hat{X}$, and $\tilde{X}$ denote the steady state level, the level deviation from the steady state, and the percentage deviation from the steady state of variable X, respectively. By iterating this equation forward and setting $\tilde{b}_{t-1} = 0$, we derive the following expression:

$$\sum_{s=t}^{\infty} \frac{\bar{g}}{(\bar{r}^b)^{s-t+1}}\tilde{g}_s = \sum_{s=t}^{\infty} \frac{\bar{T}}{(\bar{r}^b)^{s-t+1}}\tilde{T}_s - \sum_{s=t}^{\infty} \frac{\bar{b}}{(\bar{r}^b)^{s-t+1}}\hat{r}_s^b. \tag{19}$$

Finally, by using the fact that $r_t^b = \frac{1+(1-\eta)Q_t^B}{Q_{t-1}^B \pi_t}$, we can derive the following fiscal financing decomposition:

$$\underbrace{\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}}\frac{\bar{g}}{\bar{y}}\tilde{g}_s}_{\text{government consumption}} = \underbrace{\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}}\frac{\bar{T}}{\bar{y}}\tilde{T}_s}_{\text{tax revenue}} + \underbrace{\frac{\bar{b}}{\bar{y}}\hat{\pi}_t}_{\text{current inflation}} + \underbrace{\sum_{s=t+1}^{\infty} \frac{(1-\eta)^{s-t}}{(\bar{r}^b)^{s-t}}\frac{\bar{b}}{\bar{y}}\hat{\pi}_s}_{\text{future inflation}}$$
$$\underbrace{- \sum_{s=t+1}^{\infty} \frac{1-(1-\eta)^{s-t}}{(\bar{r}^b)^{s-t+1}}\frac{\bar{b}}{\bar{y}}\hat{r}_s^b}_{\text{future real interest rate}}. \tag{20}$$

See Appendix A.4 for the derivations. This equation shows that the present value of expected increases in government consumption can be decomposed into components that are financed through higher taxes, increases in current and future inflation, and decreases in future real interest rates.[12]

When the government debt has a one-period maturity ($\eta = 1$), the third term is equal to zero. Then, the present value of increases in government consumption is financed by the present value of increases in tax revenue, the increase in the inflation rate in the current period, and the discounted sum of the decreases in future real interest rate. A higher real interest rate contributes negatively to fiscal financing, as future primary surpluses are now discounted at a higher interest rate.

When the government can issue long-term debt ($\eta < 1$), increases in future inflation also contribute to fiscal financing (see the third term in equation (6)). A longer maturity of debt (a smaller η) affects the decomposition in two ways. Firstly, as indicated in the third term, it increases the weights of inflation in all periods while giving relatively greater importance to inflation in the farther future. Secondly, the fourth term demonstrates that a smaller η reduces the weights assigned to real interest rates in all future periods, as the government only needs to roll over a smaller fraction of debt in each period. Simultaneously, it also assigns a relatively higher weight to near-term real interest rates.

This decomposition allows us to study the sources of fiscal financing in the baseline economy and comparison economies in order to understand the impact of collateral constraints on fiscal financing in the optimal policy. In Section 3, we present and discuss the numerical results.

3 Quantitative analysis

We now solve the model numerically and explore its quantitative properties. We adopt a first-order approximation around the non-stochastic steady state, where the collateral constraint of high-productivity bankers strictly binds. When solving the model, we assume that their collateral constraint always binds, and we later verify that this is the case for the size of shock we consider.

[12] As there are no further shocks from date $t+1$ on, the ex-post real return on government bonds r_s^b when $s \geq t+1$ is equal to the one-period real interest rate.

3.1 Calibration

Table 1 summarizes the model's parameters. The computations are conducted at a quarterly frequency, and the discount factor β is set to 0.99. We set $\epsilon = 1$, which implies a Frisch elasticity of labor supply of one. This choice is in line with the recommendation of Chetty et al. (2011) and is appropriate for our model since it does not distinguish between the intensive and extensive margins of employment.

Table 1: Parameters

	Parameters	Value	Target/Source
Preferences			
Household discount factor	β	0.990	
Inverse intertemporal elasticity	ρ	2.000	
Disutility of labor	χ	3.400	Gertler and Karadi (2011)
Inverse Frisch elasticity	ϵ	1.000	Chetty et al. (2011)
Production Technology			
Capital share of output	α	0.283	one third of 0.850
Labor share of output	θ	0.566	two thirds of 0.850
Depreciation rate of capital	δ	0.025	
Probability of z^H	σ	0.500	
High idiosyncratic productivity	z^H	1.822	std of $\log(z_{i,t})$ is 0.3
Low idiosyncratic productivity	z^L	1.000	normalized
Collateral constraint			
Pledgeable share of capital	ξ	0.338	steady-state debt-to-GDP ratio is 61%
Government Consumption			
Gov consumption to GDP in SS	$\bar{g}/\bar{y}$	0.157	estimated
Persistence of g shock	ρ^g	0.890	estimated
Std of g shock	σ^g	1.40%	estimated

Regarding the production technology, the overall returns to scale $\alpha + \theta$ are set to 0.85, and the labor share θ is set to two-thirds of 0.85, following Midrigan and Xu (2014) and Basu and Fernald (1997). We assume a symmetric idiosyncratic productivity shock process by setting $\sigma = 0.5$, and we normalize the low realization of productivity z^L to 1. The high realization z^H is chosen such that the standard deviation of the logarithm of idiosyncratic productivity is 0.3. This value is consistent with the estimated size of TFP innovations in U.S. manufacturing firms (Asker et al., 2013).[13] In the data, the average government consumption to GDP ratio is 16% from 1948Q1 to 2022Q4. We estimate a first-order autoregressive process for aggregate government consumption using data from the same period. The standard deviation is 1.40%, and the autocorrelation is 0.89. In the baseline economy, we assume that the government issues short-term debt with a maturity of one quarter ($\eta = 1$). We examine the role played by maturities of government debt in Section 3.4.

The parameter ξ determines the tightness of the collateral constraint, which in turn affects the amount of debt the government issues in the optimal policy. We calibrate ξ in such a way that the steady-state debt-to-GDP ratio in the optimal policy matches the data (the average from 1966Q1 to 2022Q4 is 61%). In our model, the size of government debt captures both the tax-saving benefit of inflation and the cost of inflation to banks.[14] Our chosen parameter value implies that a 5% cumulative inflation causes 1.2% bank loss of net worth, while it is 3.7% in the data as documented in Cao (2019). Thus, the calibration of ξ understates the cost of inflation to the bank net worth. This conservative approach allows us to capture a lower bound on the impact of inflation on the banking sector.

In the literature on financial frictions, financial friction parameters are usually calibrated to match either the yield or yield spread of assets (Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010; Del Negro et al., 2017). In our model, our choice of ξ implies that the steady-state liquidity premium is 0.63%.[15] This value is broadly in line with estimation ins Krishnamurthy and Vissing-Jorgensen (2012), who find that the average liquidity premium from 1926 to 2008 is 0.46%.

In Appendix Section B, we show that our findings are robust to variations in the pledgeable share of capital ξ, discount factor β, inverse intertemporal elasticity of substitution ρ, capital depreciation rate δ, the probability of high idiosyncratic productivity σ, and the inverse Frisch elasticity ϵ.

[13] As shown in Asker et al. (2013), firm-level productivity exhibits persistence, with an autocorrelation coefficient of 0.8. Therefore, the standard deviation of the logarithm of productivity is $0.3/\sqrt{1-0.8^2} = 0.5$. In our model, idiosyncratic productivity is assumed to be i.i.d. We perform a conservative calibration by calibrating the standard deviation to the size of productivity innovations rather than the productivity process in the data. In the model, the larger the variance of the shock, the greater the need for capital reallocation and the demand for government bonds as collateral.

[14] In reality, banks hold other long-term nominal assets on their balance sheets, such as mortgage-backed securities and various loans. These assets also contribute to the cost of inflation for banks. However, incorporating these additional nominal assets on bank balance sheets would make the model less tractable for optimal policy analysis.

[15] The liquidity premium is defined as $4(1/\beta - \bar{r}^b)$, representing the difference between annual interest rates on government debt and an otherwise identical asset with no collateral value.

3.2 Results

3.2.1 The case of a purely transitory government consumption shock

To develop intuition, we first investigate the Ramsey optimal policy under the assumption of a purely transitory government consumption shock ($\rho^g = 0$). We adjust the standard deviation of the shock, σ^g, to match the variance of government consumption g_t in the persistent case.

Figure 2 shows the Ramsey optimal policy response to a one-standard-deviation government consumption shock in both the baseline economy and the comparison economy without collateral constraints. We equate the steady-state debt-to-GDP ratio in the comparison economy to that in the baseline model with constraints. In the comparison economy, the government maintains a relatively constant labor tax rate while increasing the inflation rate by 0.24% to reduce the real value of debt and offset the rise in government expenditure. Intuitively, higher inflation and lower real return on debt resemble a lump-sum tax on household wealth ex post, while the labor tax is distortionary, incurring convex efficiency loss. In addition, the fiscal shock leads to a crowding out of consumption and investment, while also inducing an income effect that increases labor supply and thus driving output above the steady state in the initial period.

Table 2 presents the results of fiscal financing decompositions. In the comparison economy, higher inflation in the current period finances over 124% of the present value of the increase in government consumption. The real interest rate contributes a negative 17%. Following a negative government consumption shock, consumption initially drops but eventually returns to the steady state, resulting in a higher real interest rate along this path and the negative contribution of real interest rates.

In the baseline economy with collateral constraints, relying solely on inflation to monetize outstanding debt is no longer optimal. Instead, the optimal policy response to a fiscal shock involves a combination of higher inflation and a higher tax rate, in order to balance the distortionary effect of labor tax and the cost of inflation in tightening the collateral constraint. Inflation reduces the real value of government debt, leading to a scarcity of collateral that, in turn, impedes capital reallocation and dampens TFP.

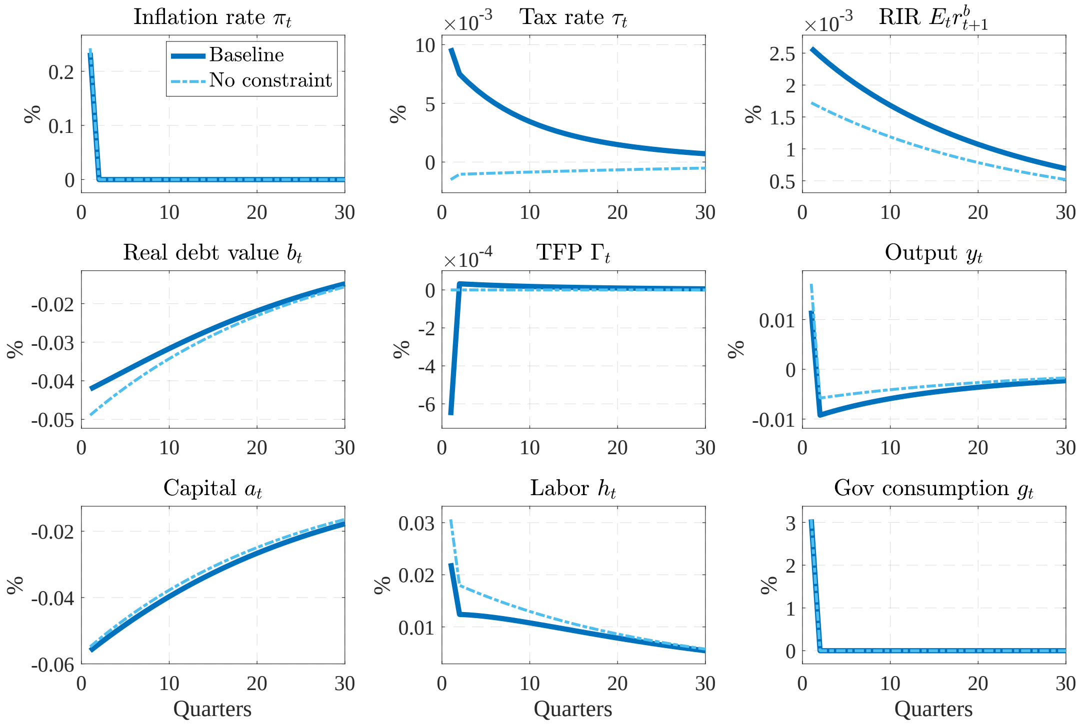

Figure 2: Optimal policy responses to a one standard-deviation government consumption shock. The shock is 3.07% with the first-order autocorrelation coefficient of 0. The solid line represents the Ramsey optimal policy in our baseline model. The solid-dashed line represents the optimal policy in an otherwise identical comparison economy but without banks' collateral constraint.

However, notice that the increase in the inflation rate is 0.23%, and its contribution to fiscal financing is 120%, only slightly smaller than their counterparts in the comparison economy. This is because the misallocation caused by inflation is only temporary in the presence of a purely transitory government consumption shock. To understand this, consider that the increase in the tax rate is persistent due to the tax smoothing motive, while the increase in government consumption is transitory, resulting in larger primary surpluses in future periods. Consequently, the policymaker can provide a larger amount of collateral $r^b_s b_{s-1}$ for periods $s \geq t+1$. Indeed, as shown in Figure 2, the TFP rises above the steady-state from period $t+1$, and the real interest rate increases by more than in the comparison economy due to the decrease in the liquidity premium.

Overall, since inflation only causes a temporary misallocation upon the impact of the shock, the Ramsey policymaker is more willing to inflate away government debt to dampen the increases in distortionary taxes. However, as we will see below, the incentive to use inflation in the optimal policy greatly declines as the government consumption shock becomes sufficiently persistent.

Table 2: Decomposition of fiscal financing

	Comparison economy with no constraint	Baseline economy
Tax revenue	-7%	5%
Current inflation	124%	120%
Real interest rate	-17%	-25%

Note: The numbers reported in the table are fractions of the present value of increases in government consumption (left-hand-side of equation (20)) financed by different sources. Each column of the table adds up to 100%.

3.2.2 The case of a persistent government consumption shock

We now set the persistence of the government consumption shock to 0.89, as indicated in Table 1, and maintain this value throughout the remainder of the paper. As shown in Figure 3, the inflation rate increases by 0.42% in response to the shock in the baseline economy, significantly smaller than the 0.86% observed in the comparison economy. In the baseline economy, the persistent increase in the government consumption reduces the primary surpluses in subsequent periods and limits the amount of debt the Ramsey policymaker can issue. Therefore, the economy endures a persistent lack of collateral and low efficiency of capital allocation, which reduces the incentive to invest in physical capital. As a result, the Ramsey policymaker is more hesitant to monetize debt and more inclined to raise the tax rate. In comparison to the economy without constraints, the baseline economy experiences greater declines or smaller increases in both capital and output.[16]

In Table 3, we observe that in the baseline economy, higher current-period inflation only finances 56% of higher government consumption, which is only half the proportion in the comparison economy (115%). Meanwhile, increases in tax revenue account for 52% of the financing. The contribution of the real interest rate is now less negative than in the comparison economy, because the reduced real debt value tightens the high-productivity bankers' collateral constraint, elevates the liquidity premium of government bonds, and consequently mitigates the rise in the real interest rate. In Appendix B.7, we study the sensitivity of the fiscal financing decomposition results to

[16] In the baseline economy, the profits of both types of bankers, ν_t^H and ν_t^L, increase above the steady state and persist at elevated levels for more than 10 quarters, resembling the qualitative pattern in aggregate labor h_t. The increase in ν_t^L is larger than that in ν_t^H, as low-productivity bankers are not subject to a binding collateral constraint.

the value of ρ^g. The contribution of inflation monotonically decreases in ρ^g, while that of taxes monotonically increases with higher values of ρ^g.

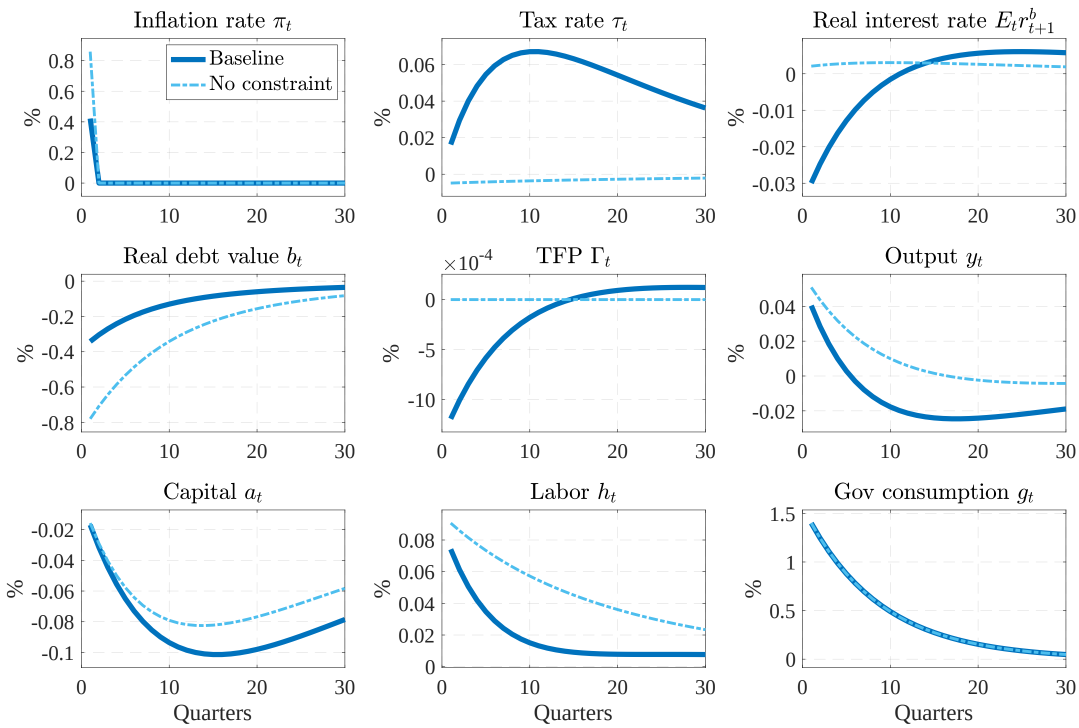

Figure 3: Optimal policy responses to a one standard-deviation government consumption shock. The shock is 1.40% with the first-order autocorrelation coefficient of 0.89. The solid line represents the Ramsey optimal policy in our baseline model. The solid-dashed line represents the optimal policy in an otherwise identical comparison economy but without banks' collateral constraint.

Table 3: Decomposition of fiscal financing

	Comparison economy with no constraint	Baseline economy
Tax revenue	-2%	52%
Current inflation	115%	56%
Real interest rate	-13%	-7%

Note: The numbers reported in the table are fractions of the present value of increases in government consumption (left-hand-side of equation (20)) financed by different sources. Each column of the table adds up to 100%.

In standard flexible-price models, Ramsey optimal policy exhibits significant inflation volatility, because state-contingent inflation serves purely as a fiscal buffer (Chari et al., 1991). Indeed, the first column of Table 4 demonstrates that in comparison economy without collateral constraints, the quarterly volatility of the labor tax rate is near zero, while the inflation rate's volatility is close to one percent on a quarterly basis. In contrast, our model, which incorporates collateral constraints, dampens the standard deviation of inflation by half, while making the labor tax rate much more volatile. Thus, our model offers a new explanation, in addition to price stickiness, for why volatile inflation is undesirable.

Table 4: Standard deviation of tax rate and inflation (quarterly)

	Comparison economy with no constraint	Baseline economy
Inflation	0.86%	0.42%
Tax rate	0.02%	0.32%

3.3 Comparison with alternative policies

The optimal policy is determined by balancing the tradeoff between tax distortions and the cost of inflation. To illustrate the tradeoff, we compare the Ramsey optimal policy with two sets of alternative policies. The first alternative policy is a constant-tax policy, where the labor tax rate τ_t is fixed at the steady-state value. In response to government consumption shocks, the government adjusts state-contingent inflation to satisfy its inter-temporal budget constraint. The second alternative policy is a zero-inflation policy where state-contingent inflation is eliminated (i.e., $\pi_t = 1$ in all periods and states) while the government retains the flexibility to optimally adjust the labor tax rate τ_t. This scenario is equivalent to one where the government issues real instead of nominal debt. The two economies with alternative policies share the same steady state as the baseline economy.

Figure 4 illustrates the optimal responses to a one-standard-deviation government consumption shock in these three economies, while Table 5 provides the decompositions of fiscal financing. In the constant-tax policy, the inflation rate increases by 0.55% in the initial period, compared to 0.42% in the optimal policy. The larger increase in the inflation rate leads to a more severe scarcity of collateral, resulting in a sharper contraction in TFP and capital investment compared to the baseline economy. Furthermore, a larger decline in the real value of government debt increases its liquidity premium, leading to a more significant decrease in the real interest rate. This larger decrease in the real interest rate helps alleviate the fiscal stress, preventing further increases in the inflation rate. As shown in Table 5, the larger decreases in the real interest rate now contributes almost 30% to the financing of fiscal shocks. However, even with this contribution, the role of inflation has surged to 72%, a significant increase from its 56% share in the baseline economy.

In the zero-inflation policy, the labor tax rate increases by 0.38% in the initial quarter and remains higher than 0.20% after 14 quarters. Consequently, labor supply is significantly depressed compared to the other two economies. The government debt increases to smooth tax distortions, leading to bankers holding more collateral, a more efficient allocation of capital among them, and ultimately a higher TFP. However, despite the improved efficiency in capital allocation, the larger increases in labor taxes lead to a more significant decline in output and capital investment compared to the other two policies. From the fiscal financing perspective, the increase in the real interest rate also exacerbates the fiscal stress. As shown in Table 5, higher real interest rate makes a substantial negative contribution of -88% to the fiscal financing.

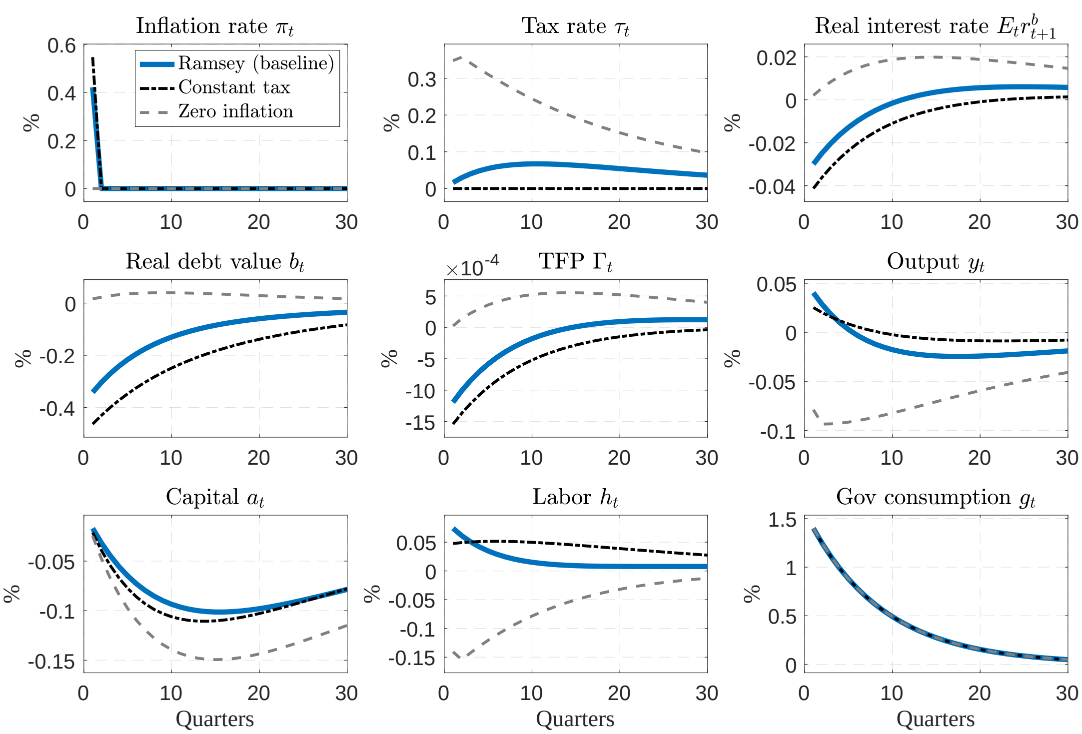

Figure 4: Optimal policy responses to a one standard-deviation government consumption shock. The solid line represents the Ramsey optimal policy, which is also depicted as the solid line in Figure 3. The dashed-dotted line represents a policy where the labor tax rate is fixed at the steady-state value. The dashed line represents a zero-inflation policy where state-contingent inflation is eliminated.

Overall, as the Ramsey planner in our baseline economy strikes a balance between tax distortions and inflation costs, the impulse responses of policy instruments and real allocations fall between those observed in the two alternative policies.

Table 5: Decomposition of fiscal financing in two alternative economies

	Constant-tax economy	Zero-inflation economy
Tax revenue	-2%	188%
State-contingent inflation	72%	0%
Real interest rate	30%	-88%

Note: The numbers reported in the table are fractions of the present value of increases in government consumption financed by different sources. Each column of the table adds up to 100%.

3.4 The role of government debt maturity

We now examine the impact of government debt maturity on the Ramsey optimal policy. When the government debt has a maturity of one period, in response to a shock, the government can only utilize inflation in the same period as the shock to adjust the real value of the debt. However, with long-term debt, the government gains the ability to adjust the real return on debt not only through current inflation but also through inflation in future periods.

We solve for the Ramsey optimal policy with various values of η, which correspond to different average government debt maturities.[17] All other parameters except η remain the same as listed in Table 1. Interestingly, in equilibrium, the responses of real allocations, real prices, and the optimal tax rate τ_t to shocks remain unchanged as the debt maturity parameter η varies, indicating that the maturity of the government debt does not alleviate the cost of inflation on banks in this environment with flexible prices. The reason behind this result is that what matters for real allocations and fiscal policy is the ex post real returns on debt r_t^b, and one-period debt ($\eta = 1$) already provides the Ramsey government with sufficient flexibility to make r_t^b state-contingent through current inflation. However, when the Ramsey government issues long-term debt ($\eta > 1$),

[17] Following the literature, the concept of Macaulay duration is used to determine the steady-state value of η given by:

$$D = \frac{1 + \bar{r}^b}{\eta + \bar{r}^b}.$$

since prices are flexible in this environment, the government is indifferent between using current or future inflation to adjust the real value of debt. As a result, the optimal responses of inflation to shocks become indeterminate. To address this issue and incentivize the Ramsey policymaker to pursue a smoother path of inflation, we introduce a small quadratic cost of inflation $\frac{\psi}{2}(\pi_t - 1)^2$ in the social resource constraint during the simulation. We examine the limiting case as ψ approaches zero by setting the parameter ψ to 1e-5. We ensure that further decreasing ψ does not affect the numerical results.[18]

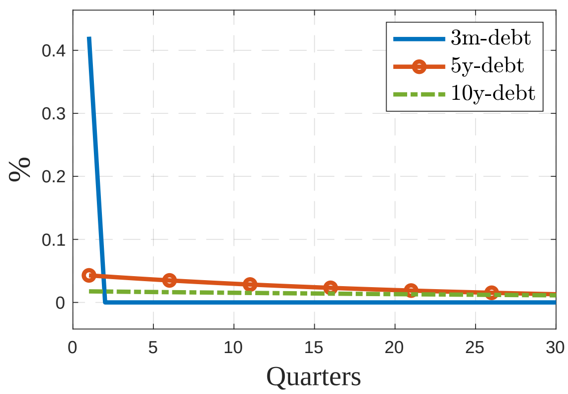

Figure 5: The optimal response of inflation to a one standard-deviation government consumption shock in economies with different average maturities of government debt

Figure 5 displays the optimal responses of inflation for three different average maturities of government debt: three months, five years, and 10 years. Note that the responses of other variables remain the same as in Figure 3, and thus are not plotted. With long-term debt, the initial response of inflation to the government consumption shock is significantly dampened, while the response also exhibits increased persistence, aligning with real-world data. When considering government debt with an average maturity of five years, which is approximately the case in the U.S., the initial increase in inflation is a mere 0.04%, in stark contrast to the 0.42% increase observed in the baseline model with one-period bonds. Moreover, considering the cumulative inflation over a 10-year period, it amounts to 0.86%, resulting in a notable reduction in the ex-post real returns on

[18] In Section 4, we develop a comprehensive sticky-price model to investigate the interaction between the maturity of government debt and price stickiness. As we will see, with sticky prices, fiscal policies and real allocations are influenced by the maturity of government debt.

debt. As the average debt maturity extends to 10 years, the initial response of inflation and the 10-year cumulative inflation decrease to 0.02% and 0.53%, respectively.

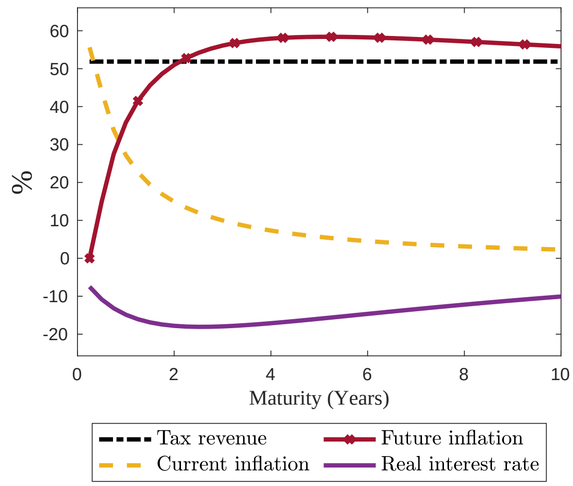

Figure 6: Fiscal financing decomposition as a function of the average government debt maturity. For each maturity, the figure plots the contributions of tax revenue, current inflation, future inflation, and real interest rate, the sum of which is 100%.

Figure 6 illustrates the changes in the fiscal financing decomposition as the average government debt maturity varies. It plots the contributions of tax revenue, current inflation, future inflation, and real interest rate against the average government debt maturity. As expected, the contribution of higher taxes remains constant since the optimal tax rate and real allocations are unaffected by the debt maturity. The contribution of higher current-period inflation monotonically decreases as the maturity lengthens. Longer maturities result in a more subdued initial response of inflation, leading to its reduced contribution to fiscal financing. For example, when the average maturity is five years, the contribution of higher current inflation drops to 6%. This contribution further decreases to 2% when the debt has an average maturity of 10 years. Interestingly, the contribution of higher future inflation exhibits a non-monotonic pattern due to two competing forces. On the one hand, longer debt maturities lead to more persistent inflation responses, which tends to increase the contribution of future inflation. On the other hand, the inflation process exhibits mean reversion. But as equation (20) shows, a longer maturity (smaller η) assigns relatively a larger weight to inflation in the farther future, reducing the contribution of future inflation. The overall contribution of inflation (the sum of higher current and future inflation contributions) is 64% for a five-year debt maturity and 58% for a 10-year debt maturity.[19]

Overall, our analysis illustrates that long-term government debt enables the Ramsey policy maker to leverage inflation in future periods to reduce the real value of debt. In the next section, we will explore how the presence of long-term debt helps alleviate the cost of inflation imposed by price stickiness. In a flexible-price environment, the maturity of government debt fails to mitigate the cost of inflation on banks, as we have discussed. It does not influence real allocations and real prices in that environment.

4 Introducing price stickiness

As another cost of inflation, price stickiness imposes significant costs on abrupt changes in the price level, limiting the extent to which state-contingent inflation can be effectively used in optimal fiscal and monetary policy. In this section, we augment the baseline model with price stickiness and study how it curtails the use of inflation in the optimal policy. In addition, we explore to what extent this reduction in inflation usage can be mitigated by introducing long-term government debt.

4.1 Model

In order to incorporate price stickiness into the model, we introduce a continuum of retail firms operating as monopolistic competitors. These retail firms purchase wholesale goods from the competitive firms owned by bankers. The retail firms differentiate these goods at no cost and then sell them to households. Due to their monopoly power, the retail firms can set prices above their marginal costs. The profits generated from their retail activity are then rebated lump-sum to households.[20] The final goods purchased by households for consumption and investment are aggre-

[19] The fiscal financing decomposition also reveals a non-monotonic pattern in the contribution of the changes in real interest rate. It is important to note that the government debt maturity does not impact the impulse response of the real interest rate. As shown in Figure 3, the real interest rate initially drops below the steady state but eventually rises above it. In light of Equation (20), a longer maturity (smaller η) tends to bring the weights assigned to real interest rates in all periods closer to zero, making their contributions less negative. However, a longer maturity also reduces the weight assigned to near-term real interest rates, which are below the steady state, consequently leading to a more negative contribution of the real interest rate.

[20] The model follows the approach of Bernanke et al. (1999) in distinguishing between the competitive and flexible-price firms held by bankers from the sticky-price retail firms. Directly introducing price stickiness to the firms held by bankers would significantly complicate the tractability of the model, as these firms experience idiosyncratic

gated from the differentiated goods using a constant elasticity of substitution function. Households choose their demand for different types of goods, denoted by j:

$$y_{j,t} = y_t \left(\frac{P_{j,t}}{P_t}\right)^{-\nu}, \tag{21}$$

where ν represents the elasticity of substitution across goods sold by retail firms. P_t denotes the aggregate nominal price level, and $P_{j,t}$ denotes the nominal price of type-j good at time t.

To incorporate price stickiness into the model, we introduce a Rotemberg-style price adjustment cost. When a retail firm j needs to adjust its nominal price $P_{j,t}$, it incurs a cost equal to $\frac{\psi}{2}\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2$ units of final goods. Retail firm j sets price $\{P_{j,s}\}_{s\geq t}$ to maximize the expected discounted sum of real profits that it rebates to the household:

$$\max_{P_{j,s}} \quad \mathbb{E}_t \sum_{s\geq t} \Lambda_{t,s} v_s^R \equiv \mathbb{E}_t \sum_{s\geq t} \Lambda_{t,s} \left[\frac{P_{j,s}}{P_s} y_{j,s} - m_s y_{j,s} - \frac{\psi}{2}(\frac{P_{j,s}}{P_{j,s-1}} - 1)^2\right],$$

subject to the demand function for good j in equation (21). $\Lambda_{t,s}$ is the household's real stochastic discount factor, and m_t is the real price (in the units of final goods) to purchase wholesale goods from bankers' firms. In other words, it is the real marginal cost for a retailer to produce differentiated goods j.

We focus on a symmetric equilibrium where each retail firm j sets the same price $P_{j,t}$ and $P_{j,t} = P_t$ for all j. The optimality condition of retail firms takes the form of the New Keynesian Phillips curve:[21]

$$\left[\nu m_t - (\nu - 1)\right] y_t - \psi \left[(\pi_t - 1)\pi_t - \mathbb{E}_t \Lambda_{t,t+1} \left(\pi_{t+1} - 1\right) \pi_{t+1}\right] = 0. \tag{22}$$

The parameter ψ captures the degree of price stickiness, and when prices are fully flexible ($\psi = 0$), equation (22) simplifies to $m_t = \frac{\nu-1}{\nu}$.

productivity shocks. In such a setup, high-productivity firms would set lower prices and vice versa, necessitating the tracking of price history and cross-sectional price distributions. By separating these two types of firms, the model maintains tractability while still capturing the essential price stickiness in the retail sector.

[21] Log-linearizing equation (22), we get the more commonly used New Keynesian Phillips curve:

$$\tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \frac{\nu - 1}{\psi} \bar{y} \tilde{m}_t,$$

where $\tilde{\pi}_t$ denotes the percentage deviation of inflation rate from the steady state, and $\bar{y}$ represents the steady-state value of output.

The remaining aspects of the model follow the same structure as the baseline model discussed in Section 2 and are outlined in Appendix Section A.5. The competitive equilibrium, the Ramsey optimal policy, and the fiscal financing decomposition are also defined in a similar manner.

4.2 Numerical analysis

This sticky-price model introduces two new parameters, namely the elasticity of substitution ν and the degree of price stickiness ψ. We set these parameters to values estimated from U.S. data in Christiano et al. (2005). In line with their findings, we set $\nu = 6$, which corresponds to a markup of 20% for retail firms. For the price stickiness parameter ψ, we set it such that in a linearized setup, it replicates the slope of the Phillips curve derived using the Calvo price-setting model with an average duration of prices lasting three quarters.[22] To maintain consistency with the baseline model, we recalibrate the tightness parameter of the collateral constraint ξ such that the steady-state debt-to-GDP ratio remains at 61%. All other parameters retain the same values as specified in the baseline model (refer to Table 1).

In the steady state of the Ramsey policy, the price inflation rate is zero ($\bar{\pi} = 1$). This result is intuitive because there are no benefits to having inflation in the non-stochastic steady state where no fiscal shocks are present. On the other hand, any departure from zero inflation incurs a cost in real resources.

Figure 7 illustrates the optimal policy response to a government consumption shock, while Table 6 presents the fiscal financing decomposition in economies with different average government debt maturities, all subject to collateral constraints. In the economy where the government bond has a maturity of one period (3 months), the government can only increase the inflation rate in the current period to reduce the real return on debt, but it is costly to do so due to the presence of the price adjustment cost. As a result, the inflation rate rises by only 0.11% in the same quarter as the fiscal shock occurs, compared to 0.42% in the baseline economy.[23] Consequently, the contribution

[22] The Calvo model suggests that the slope of the Phillips curve is given by $\frac{(1-\kappa)(1-\beta\kappa)}{\kappa}$, where κ is the probability of not being able to re-optimize price (Galí, 2009). $\kappa = 0.667$ corresponds to an average price duration of three quarters. In the Rotemberg-style sticky-price model, the slope of the Phillips curve is $\frac{(\nu-1)\bar{y}}{\psi}$, where $\bar{y}$ is the steady-state value of output. By setting $\frac{(\nu-1)\bar{y}}{\psi} = \frac{(1-\kappa)(1-\beta\kappa)}{\kappa}$, we ensure consistency between the two models.

[23] Inflation in the baseline flexible-price economy is not plotted for presentation purposes, as its magnitude is significantly larger than in the other three economies. Instead, interested readers can refer to Figure 4 for the impulse response of inflation in the flexible-price economy.

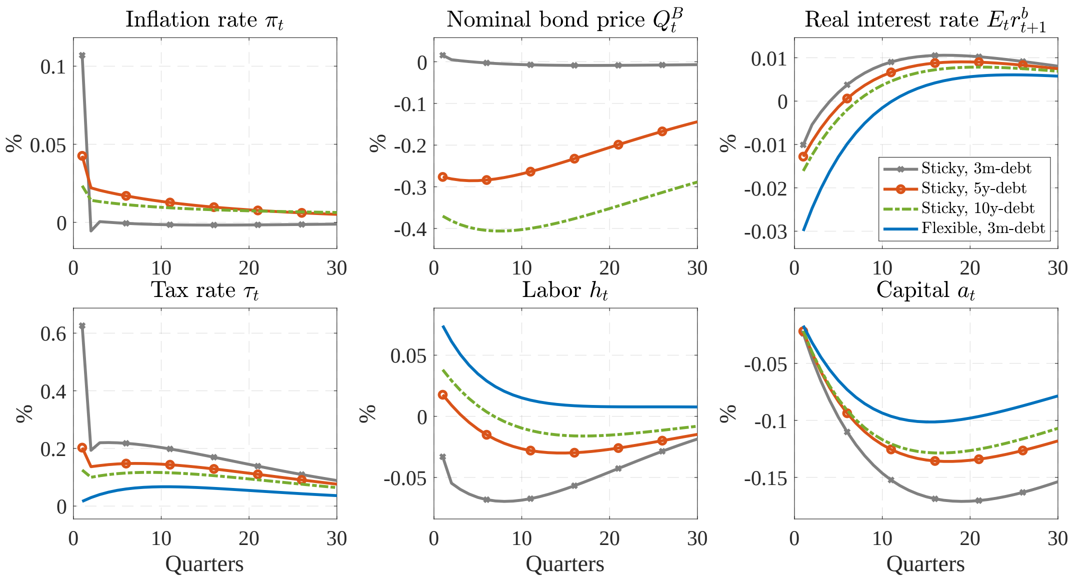

Figure 7: Optimal policy responses to a one standard-deviation government consumption shock. The shock is 1.40% with the first-order autocorrelation coefficient of 0.89. All economies are associated with collateral constraints. The three sticky-price economies only differ in the average maturity of government debt. The flexible-price economy is the same as in the baseline model in Figure 4, where the government issues short-term debt.

of higher current inflation reduces to 14%, in contrast with 56% in the baseline economy. As the government monetizes less of its debt, more collateral is present in the economy, resulting in a lower liquidity premium and higher interest rate of government debts. Therefore, the negative contribution of the real interest rate becomes more pronounced at -44%. The smaller contributions of inflation and the more negative contribution of the real interest rate imply a significantly larger reliance on taxes. Overall, this analysis aligns with the previous studies highlighting that price stickiness significantly limits the use of inflation in optimal policies if the government debt has a short maturity.

Table 6: Decomposition of fiscal financing

	Sticky price 3-month debt	Sticky price 5-year debt	Sticky price 10-year debt
Tax revenue	130%	98%	86%
Current inflation	14%	5%	3%
Future inflation	0%	26%	33%
Real interest rate	-44%	-29%	-22%

Note: All economies are associated with collateral constraints and price stickiness. They only differ in the average maturity of government debt.

With long-term government debts, inflation assumes a much larger role in the optimal policy. Compared to the case of short-term debt, the optimal response of inflation in the scenario with five-year debt is significantly smaller in the initial period (0.04%), but it exhibits much more persistence in subsequent periods. Over the 10-year period following the fiscal shock, cumulative inflation amounts to 0.40%. The sustained decrease in the inflation rate leads to a 0.28% decline in the nominal bond price at the time of the shock, thereby facilitating a reduction in the real debt return. The contributions of higher current and future inflation sum up to 31%, contrasting with the 14% observed in the short-term debt economy. Notably, future inflation plays a more substantial role relative to current inflation (26% versus 5%). Additionally, the negative contribution of the real interest rate diminishes, as the real interest rate experiences a smaller increase and the government only rolls over a fraction of the debt when the debt is long-term. The ability to monetize government debt also enables the government to significantly mitigate the rise in the tax rate. In terms of real allocations, the economy with five-year government bonds align more closely with the baseline flexible-price economy, as shown in Figure 7.

The contrast between the short-term debt economy and the 10-year debt economy becomes more pronounced. Inflation exhibits a smoother and more persistent pattern, with an initial response of 0.02% and a cumulative 10-year inflation of 0.34%. The overall contribution of inflation is even more substantial in the long-term debt case, amounting to 36%. Moreover, the tax rate, real interest rate, and real allocations become even closer to those observed in the flexible-price economy.

These exercises show that long-term government debt significantly mitigates the costs associated with using inflation to adjust the real value of debt in the presence of price stickiness. The long maturity of debt, however, does not reduce the cost of inflation for banks, as previously observed in Section 3.4.

5 Applications

How much inflation should a government optimally generate when it receives exogenous spending shocks observed in the data? In this section, we use the model to study the financing of the wars in Afghanistan, Iraq and Syria by the U.S. government, as well the financing of escalating government expenditures in the COVID-19 pandemic.

5.1 War financing

The total appropriations for the wars in Afghanistan, Iraq and Syria by the U.S. government from 2001 to 2021 amounted to \$2.05 trillion (Crawford, 2021). The left panel of Figure 8 presents the appropriations for wars as a percentage of total government consumption. As depicted, the budgetary costs of the wars exceeded 7% of total government consumption at their peak.

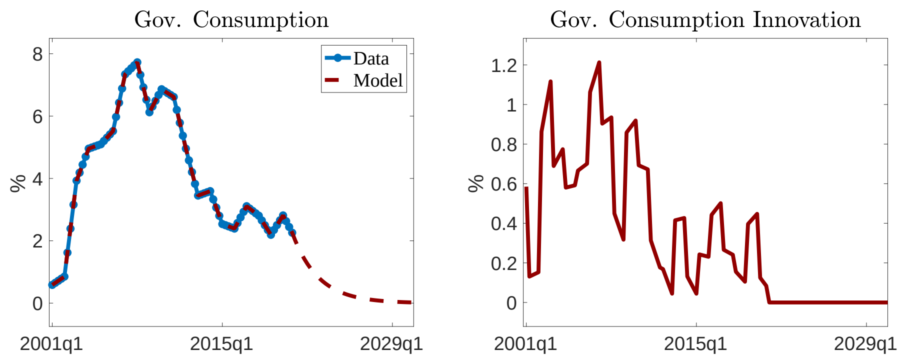

Figure 8: The left panel plots the percentage increase in government consumption resulting from wars in Afghanistan and Iraq, and Syria. The right panel plots the series of innovations to the government consumption process that aligns it with the data presented in the left panel.

We use our model to study optimal fiscal and monetary policy response to increases in war appropriations. We simulate the model with government consumption shocks that correspond to observed war costs in the data. To do this, we assume that the government consumption process follows the AR(1) process we estimated in the previous sections. We then calculate the series of shocks that align the government consumption process with the war appropriation data from 2001 to 2021. We assume that beyond 2021, no further shocks occur, and government consumption decreases according to the rate in the AR(1) process. The right panel in Figure 8 illustrates the constructed series of shocks applied to the government consumption process.

We calibrate the model to match the characteristics of the U.S. economy prior to the wars. In 2000, the debt-to-GDP ratio stood at 55%. Additionally, according to the Organisation for Economic Co-operation and Development, the average maturity of government debt was 5.8 years. We assume that the economy was in the steady state prior to the arrival of the series of war shocks.

Figure 9 presents the policy recommendations from three different models regarding war financing as well as policy variables in the data. In the model without collateral constraints or price stickiness, the government relies solely on inflation to adjust the real debt value, while the tax rate remains relatively stable. Over a period of 25 years, the average increase in inflation rate is 1.98% annually. In the economy with collateral constraints but flexible price, the presence of financial friction reduces the increase in inflation rate, resulting in an average increase of 1.13% annually. Additionally, the government raises the labor tax rate by an average of 0.62 percentage points. Lastly, in the economy with both collateral constraints and price stickiness, the effect of price stickiness further diminishes the reliance on inflation, with an average increase of only 0.50% annually. Consequently, the labor tax rate experiences a larger rise, averaging 1.31 percentage points.

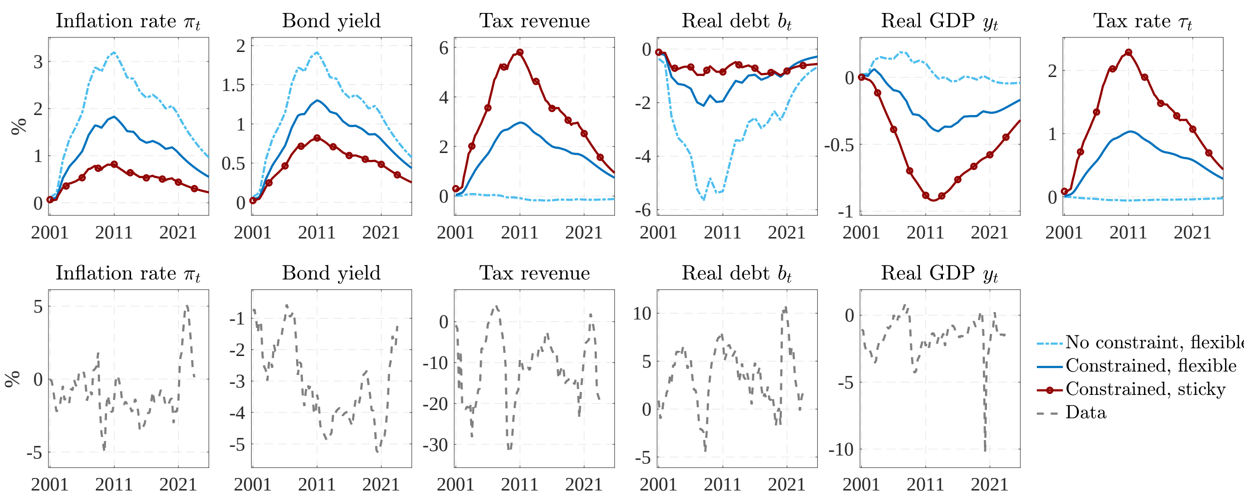

Figure 9: War financing. The top panels illustrate the optimal policy responses in the models, while the bottom panels depict the dynamics of policy variables in the data. The inflation rate and bond yield in the models are annualized. In the data, inflation rate, bond yield, and tax revenue are measured by CPI inflation, 5-year Treasury yield, and federal government tax receipts, respectively. We apply the natural logarithm to the data on real debt, tax revenue, and real GDP, and then apply the HP filter. All data are presented as changes from 2000Q4.

In the data, following the start of these wars, the inflation rate did not exhibit a noticeable increase, and federal government tax receipts fell below the trend due to the implementation of the Bush tax cuts. Consequently, real debt rose above trend. In Appendix Section C, we run a VAR model to investigate the impact of war spending on fiscal and monetary policy. We find some evidence that a positive shock to the national defense expenditures results in an increase in the real debt. In addition, it induces an immediate decrease in the inflation rate, followed by an increase after a few quarters.

Empirically analyzing the impact of wars in Afghanistan, Iraq and Syria on policy variables is complex, given the modest increase in defense expenditures associated with these wars. Martin (2012) shows that the Civil War and two World Wars, which involved substantial increases in government expenditure, were associated with contemporaneous increases in debt, tax revenue and inflation. Their model successfully replicates these empirical regularities. Our model recommends increases in inflation and tax revenues but suggests a decrease in real debt. The recommendation of a decrease in real debt aligns with the common result of Ramsey optimal fiscal and monetary policy models (e.g., Lucas and Stokey, 1983; Chari et al., 1991), as monetary policy effectively makes government debt more state-contingent.

5.2 Financing of federal government expenditures in the COVID-19 pandemic

In response to the COVID-19 pandemic, the federal government provided direct support for the healthcare response and took measures to address the economic impact of the pandemic. As illustrated in Figure 10, expenditures on public health activities and the total expenditures of the federal government increased sharply in 2020.

We examine our model's recommendations on how the government should finance the increased expenditures. To capture the overall magnitude of government expenditure increases, we use the rise in federal government expenditures as the empirical measure of the increase in g_t, recognizing that some COVID-19-related government expenditures take the form of transfer payments rather than government consumption. The innovations to g_t are constructed in a similar way as in the previous subsection. We calibrate the model to match the characteristics of the U.S. economy at the end of 2019: the debt-to-GDP ratio stood at 106%, and the average maturity of government debt was 6.2 years (De Graeve and Mazzolini, 2023).

Figure 11 presents optimal policy responses as well as the dynamics of policy variables in the data. In the model with both collateral constraints and price stickiness, inflation would peak in 2020Q2 and 2021Q1 as government expenditure soared. The average increase in the optimal inflation rate from 2020 to 2023 is 1.90%, while it is 2.30% and 3.28% in the flexible price model with collateral constraints and in the model without collateral constraints. In comparison, inflation in the data peaked in 2022Q2, and the rise in bond yield and tax revenue also occurred later. All models recommend declines in real debt, while in the data it increased. Our models focus solely on the public-expenditure aspect of the COVID-19 pandemic and do not incorporate other factors

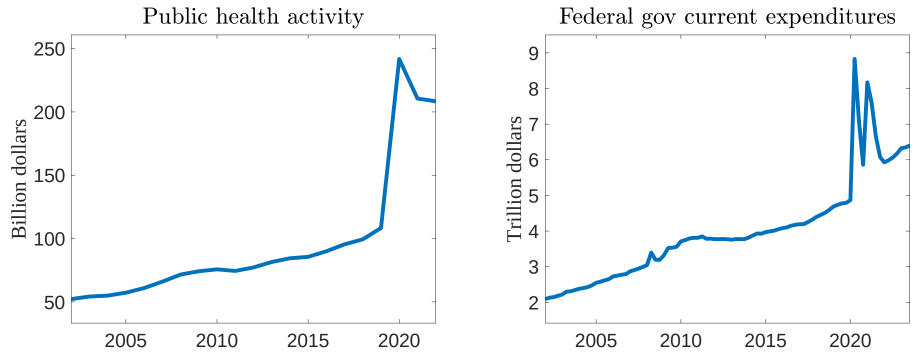

Figure 10: The left panel plots expenditures on public health activity. The right panel plots federal government current expenditures. Data sources: Centers for Medicare & Medicaid Services and Federal Reserve Economic Data.

such as lockdowns and supply chain disruptions. Therefore, they do not account for the sharp decline in GDP observed in the data.

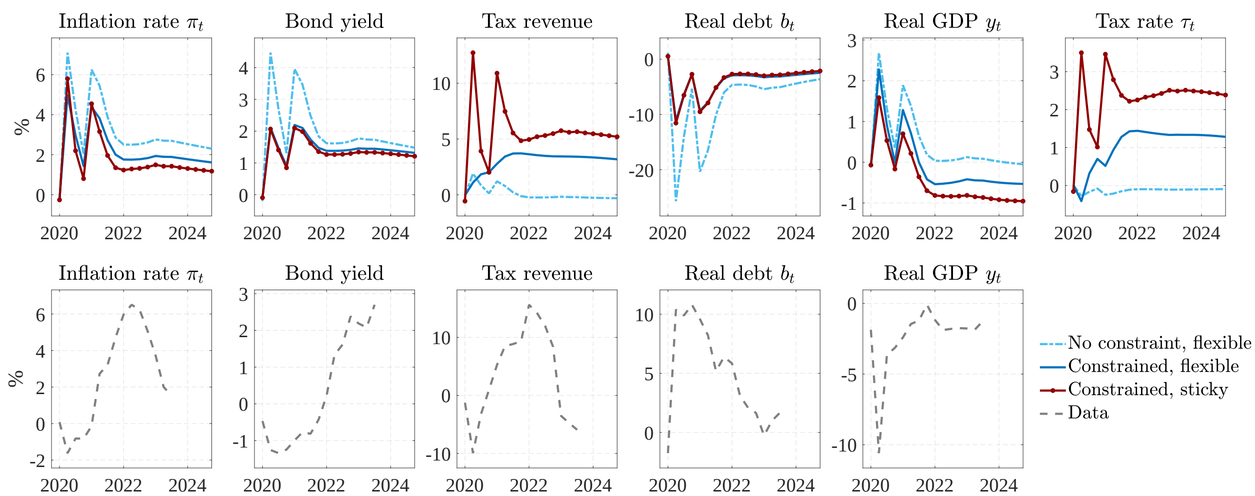

Figure 11: Financing of increased government expenditures in the COVID-19 pandemic. The top panels illustrate the optimal policy responses in the models, while the bottom panels depict the dynamics of policy variables in the data. The inflation rate and bond yield in the models are annualized. In the data, inflation rate, bond yield, and tax revenue are measured by CPI inflation, 5-year Treasury yield, and federal government tax receipts, respectively. We apply the natural logarithm to the data on real debt, tax revenue, and real GDP, and then apply the HP filter. All data are presented as changes from 2019Q4.

6 Conclusion

In this paper, we study Ramsey optimal fiscal and monetary policy in the presence of the cost of inflation on banks through banks' holdings of nominal claims. The model captures the impact of inflation on the banks' net worth and the consequent tightening of their collateral constraints. We study how a Ramsey policymaker navigates the trade-off between the distortionary effect of taxes and the cost of inflation on banks.

Our main finding is that, quantitatively, inflation plays a much smaller role in fiscal financing compared with standard models, and it is also much less volatile. Thus, we underscore the substantial impact of the bank balance sheet costs associated with inflation on the optimal fiscal and monetary policy.

We also introduce price stickiness and a long maturity of government debt into the model. The optimal response of inflation becomes modest and persistent. The maturity of government debt has a large impact on the optimal policy. As the maturity of government debt lengthens, the role of inflation in optimal fiscal financing becomes more prominent and effective. However, long-term government debt does not mitigate the cost of inflation on banks.

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Appendix

A Characterizations and proofs

A.1 Characterization of the aggregate economy

Let $x_t = \frac{k_t^H}{a_{t-1}}$ represent the amount of capital used by the high-productivity banks as a fraction of the aggregate capital stock. Then the following relationship hold:

$$\frac{k_t^L}{a_{t-1}} = \frac{1-\sigma x_t}{1-\sigma}.$$

As there is no friction in the labor market, the marginal product of labor is equalized between high- and low-productivity banks:

$$\theta z^H (k_t^H)^\alpha (n_t^H)^{\theta-1} = \theta z^L (k_t^L)^\alpha (n_t^L)^{\theta-1}.$$

By solving for the ratio of employment at the two types of banks, n_t^H/n_t^L, from the above equation and considering the labor market clearing condition $\sigma n_t^H + (1-\sigma)\, n_t^L = h_t$, we can obtain the fraction of labor used by each type of banks:

$$\frac{n_t^H}{h_t} = \frac{(z^H)^{\frac{1}{1-\theta}} (x_t)^{\frac{\alpha}{1-\theta}}}{\Gamma(x_t)^{\frac{1}{1-\theta}}}, \tag{A.1}$$

$$\frac{n_t^L}{h_t} = \frac{(z^L)^{\frac{1}{1-\theta}} \left(\frac{1-\sigma x_t}{1-\sigma}\right)^{\frac{\alpha}{1-\theta}}}{\Gamma(x_t)^{\frac{1}{1-\theta}}}. \tag{A.2}$$

Given the allocations of capital $\left(k_t^H/a_{t-1}, k_t^L/a_{t-1}\right)$ and the allocations of labor $\left(n_t^H/h_t, n_t^L/h_t\right)$, the aggregate production function can be written as:

$$\begin{aligned} y_t &= \sigma z^H (k_t^H)^\alpha (n_t^H)^\theta + (1-\sigma) z^L (k_t^L)^\alpha (n_t^L)^\theta \\ &= \left[\sigma z^H (\frac{k_t^H}{a_t})^\alpha (\frac{n_t^H}{h_t})^\theta + (1-\sigma) z^L (\frac{k_t^L}{a_t})^\alpha (\frac{n_t^L}{h_t})^\theta\right] a_t^\alpha h_t^\theta \equiv \Gamma(x_t) a_{t-1}^\alpha h_t^\theta. \end{aligned}$$

In first order conditions (8) and (9), we can substitute for k_t^H, k_t^L, n_t^H, and n_t^L using the following expressions: $k_t^H = x_t a_{t-1}$, $k_t^L = \frac{1-\sigma x_t}{1-\sigma} a_{t-1}$, and equations (A.1) and (A.2). By doing so, we can derive w_t, q_t, and μ_t^H as functions of a_{t-1}, h_t, and x_t.

A.2 Proof of the equivalence between the primal approach and the Ramsey problem

Proof of the "only if". Our goal is to establish that the set of competitive equilibrium conditions implies the set of conditions in the primal approach. The latter includes the social resource constraint (2), the flow implementability constraint (17), the Euler equation (14), the collateral constraint (18), $\boldsymbol{\mu}^{\mathbf{H}}(.) \geq 0$, and the complementary slackness condition. It is obvious that the social resource constraint (2), the Euler equation (14), $\boldsymbol{\mu}^{\mathbf{H}}(.) \geq 0$, and the complementary slackness condition are satisfied by the competitive equilibrium conditions. It remains to show that the implementability constraint (17) and the collateral constraint (18) also hold.

Implementability condition. Using equation (7) to substitute for $(1-\tau_t)w_t h_t$ in the household budget constraint (5), we obtain:

$$c_t + a_t + b_t = \left[\sigma v_t^H + (1-\sigma)v_t^L\right] - \frac{U_{h,t}}{U_{c,t}} h_t + q_t a_{t-1} + r_t^b b_{t-1}.$$

From the definition of the profit function (6), we get:

$$\sigma v_t^H + (1-\sigma)v_t^L = y_t - w_t h_t - [q_t - (1-\delta)]a_{t-1}.$$

The labor and capital demand conditions (8)-(9) imply:

$$w_t h_t = \theta y_t,$$

and

$$[q_t - (1-\delta)]\, a_{t-1} = \alpha y_t - \sigma \mu_t^H k_t^H = \alpha y_t - \frac{\sigma \mu_t^H}{q_t - \xi}\left(r_t^b b_{t-1} + q_t a_{t-1}\right). \tag{A.3}$$

The last step in equation (A.3) holds regardless of whether the collateral constraint for the high-productivity bankers binds or not.[24] Due to frictions in capital allocations, the share of capital

[24] To see this, if the collateral constraint binds for the high-productivity bankers, we have:

$$k_t^H = \frac{1}{q_t - \xi}\left(r_t^b b_{t-1} + q_t a_{t-1}\right).$$

income measured at market price of capital q_t is smaller than α when the collateral constraint strictly binds for the high-productivity bankers (i.e., $\mu_t^H > 0$).

The total profits of firms can be expressed as:

$$\sigma v_t^H + (1-\sigma) v_t^L = (1-\alpha-\theta)\, y_t + \frac{\sigma \mu_t^H}{q_t - \xi} \left(r_t^b b_{t-1} + q_t a_{t-1} \right).$$

Now, we use the above expression to substitute for total profits $\sigma v_t^H + (1-\sigma) v_t^L$ in the household budget constraint. This yields:

$$\begin{aligned} &[U_{c,t} c_t + U_{h,t} h_t - U_{c,t}(1-\alpha-\theta) y_t] + U_{c,t}\,(b_t + a_t) \\ &= U_{c,t} \left(1 + \frac{\sigma \mu_t^H}{q_t - \xi}\right) \left(r_t^b b_{t-1} + q_t a_{t-1} \right). \end{aligned} \tag{A.4}$$

Taking conditional expectation in date $t-1$ on both sides of the equation (A.4) and using the Euler equations (14) and (15), we arrive at the flow implementability condition in equation (17).

Collateral constraint. By combining the government budget constraint (3) and equation (16), we can express the outstanding value of debt at the beginning of period t by:

$$r_t^b b_{t-1} = \theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t.$$

Substituting for $r_t^b b_{t-1}$ in the collateral constraint using the equation above, we obtain the form of the collateral constraint as shown in equation (18).

Proof of the "if". Our goal is to show that if allocations $\{a_t, h_t, x_t, c_t, b_t\}_{t\geq 0}$ satisfy the set of constraints in the primal approach (equations (2), (17), (14), (18)), we can construct output y_t, prices $\{w_t, q_t, \mu_t^H\}_{t\geq 0}$ and policies $\{\tau_t, r_t^b\}_{t\geq 0}$ that satisfy all the competitive equilibrium conditions (equations (3)–(5), (7), (10)–(15)).

The wage rate w_t, the price of capital q_t, the multiplier on high-productivity bankers' collateral constraint μ_t^H, and the tax rate τ_t are determined by equations (11), (12), (13), and (16), respectively. The real return r_t^b can be obtained by rearranging the government budget constraint

If the collateral constraint does not bind, we have $\mu_t^H = 0$. In either case, equation (A.3) holds.

(3):

$$r_t^b = \frac{(1-\tau_t)\, w_t h_t + b_t - g_t}{b_{t-1}}. \tag{A.5}$$

It is straightforward to verify that the allocations $\{a_t, h_t, x_t, c_t, b_t\}_{t\geq 0}$, prices $\{w_t, q_t, \mu_t^H\}_{t\geq 0}$, and policies $\{\tau_t, r_t^b\}_{t\geq 0}$ satisfy competitive equilibrium conditions (3), (7), (11), (12), (13), and (14). Output y_t can be constructed to satisfy equation (10). The household budget constraint (5) is satisfied due to the Walras's Law. We can also show that the collateral constraint (4) holds by substituting (A.5) into equation (18).

It remains to show that the Euler equation (15) holds. First, following the same steps in the "only if" part, one can show that equation (A.4) holds. Then, by taking the conditional expectation of equation (A.4), applying the Euler equation (14) to it, and comparing it with the implementability condition (17), one can show that the Euler equation (15) holds.

A.3 Ramsey problem in the comparison economy without collateral constraints

In the comparison economy without collateral constraints, a household's decision problem is to choose $\{k_t^s, n_t^s, h_t, c_t, a_t, B_t\}_{t=0}^{\infty}$ to maximize utility (1), subject to the end-of-period budget constraint (5).

The set of first-order conditions is similar to that in the problem with the collateral constraint, and the aggregate economy can be characterized in the same manner. The main difference is that capital allocations between the two types of bankers are always optimal; in other words, $x_t = x^*$.

Given the initial household asset positions a_{-1} and B_{-1}, and the process of government consumption shocks $\{g_t\}_{t\geq 0}$, the *competitive equilibrium* can be summarized by a set of allocation $\{y_t, a_t, h_t, c_t, B_t\}_{t\geq 0}$, prices $\{q_t, w_t, P_t\}_{t\geq 0}$, and fiscal and monetary policies $\{\tau_t, Q_t^B\}_{t\geq 0}$ satisfying (3), (5), (7), (10)-(12), and (14)-(15); x_t is set to x^* and μ_t^H is set to 0 when they show up in these equations.

We can follow the steps outlined in Appendix A.2 to show the equivalence of the competitive equilibrium conditions to the conditions in the primal approach.

A.4 Derivations of the fiscal financing decompositions

In this subsection, we derive equation (20). The period-t real return on government bonds is given by:

$$r_t^b = \frac{1 + (1-\eta) Q_t^B}{Q_{t-1}^B \pi_t}. \tag{A.6}$$

By linearizing this equation and using the fact that $\tilde{Q}_{t-1}^B = 0$, we have:

$$\hat{r}_t^b = \frac{1-\eta}{\bar{\pi}} \tilde{Q}_t^B - \frac{\bar{r}^b}{\bar{\pi}} \hat{\pi}_t,$$

which shows that the real return on debt depends on the nominal bond price and the inflation rate in the current period. The nominal bond price Q_t^B is, in turn, a function of the future real interest rates and inflation rates, which can be derived by iterating $\tilde{Q}_t^B$ forward using a linearized version of equation (A.6):

$$\tilde{Q}_t^B = \sum_{s=t+1}^{\infty} \left(\frac{1-\eta}{\bar{r}^b \bar{\pi}} \right)^{s-t-1} \left(-\frac{\hat{\pi}_s}{\bar{\pi}} - \frac{\hat{r}_s^b}{\bar{r}^b} \right).$$

Therefore, we can express the ex-post real return r_t^b as a function of current and future inflation rates and future real interest rates:

$$\hat{r}_t^b = \sum_{s=t+1}^{\infty} \frac{(1-\eta)^{s-t}}{(\bar{r}^b \bar{\pi})^{s-t-1}} \left(-\frac{\hat{\pi}_s}{\bar{\pi}} - \frac{\hat{r}_s^b}{\bar{r}^b \bar{\pi}} \right) - \frac{\bar{r}^b}{\bar{\pi}} \hat{\pi}_t.$$

By combining this equation with equation (19) and using the fact that $\bar{\pi} = 1$, we arrive at the fiscal financing decomposition condition (20):

$$\begin{aligned}
\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{g}}{\bar{y}} \tilde{g}_s &= \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{y}} \tilde{T}_s - \frac{\bar{b}}{\bar{y}} \frac{\hat{r}_t^b}{\bar{r}^b} - \sum_{s=t+1}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{b}}{\bar{y}} \hat{r}_s^b \\
&= \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{y}} \tilde{T}_s + \frac{\bar{b}}{\bar{y}} \hat{\pi}_t + \sum_{s=t+1}^{\infty} \frac{(1-\eta)^{s-t}}{(\bar{r}^b)^{s-t}} \frac{\bar{b}}{\bar{y}} \hat{\pi}_s \\
&\quad - \sum_{s=t+1}^{\infty} \frac{1-(1-\eta)^{s-t}}{(\bar{r}^b)^{s-t+1}} \frac{\bar{b}}{\bar{y}} \hat{r}_s^b.
\end{aligned}$$

A.5 Model with sticky prices

In this subsection, we describe the model with sticky prices analyzed in Section 4.

A household's decision problem is to choose $\{k_t^s, n_t^s, h_t, c_t, a_t, B_t\}_{t=0}^{\infty}$ to maximize utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho} - 1}{1-\rho} - \chi \frac{\int_0^1 h_{j,t}^{1+\epsilon} dj}{1+\epsilon} \right),$$

subject to the end-of period budget constraint

$$c_t + a_t + \frac{Q_t^B B_t}{P_t} = \left[\sigma v_t^H + (1-\sigma) v_t^L\right] + \nu_t^R + (1-\tau_t)\, w_t h_t + \frac{1+(1-\eta)Q_t^B}{P_t} B_{t-1} + q_t a_{t-1},$$

and the collateral constraint

$$k_{i,t} \leq \frac{1}{q_t - \xi} \times \left[q_t a_{t-1} + \frac{1+(1-\eta)Q_t^B}{P_t} B_{t-1} \right],$$

where $v_t^s = m_t z^s F\left(k_t^s, n_t^s\right) - w_t n_t^s - \left[q_t - (1-\delta)\right] k_t^s$, $s \in \{H, L\}$. The households' problem differs from the baseline model in Section 2 in two ways. First, as the bankers' firms sell their products to retail firms, they face a price of m_t instead of one. m_t is the real price of bankers' goods sold to retailers. Second, the households also receive the profits made by the retail firms (term ν_t^R in the budget constraint).

Retail firms are monopolistic competitors. They purchase goods from competitive firms owned by bankers, differentiate these goods at no cost, and then resell them to households. We assume that profits from retail activity are rebated lump-sum to households.

The final goods used in household consumption and investment are aggregated from the differentiated goods using constant elasticity of substitution (CES) technology, represented by the function:

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}.$$

The optimal quantity of each variety $y_{j,t}$ is chosen by solving the following maximization problem:

$$\max_{y_{j,t}} \quad P_t y_t - \int_0^1 P_{j,t} y_{j,t} dj,$$

which generates the demand function for variety j:

$$y_{j,t} = y_t \left(\frac{P_{j,t}}{P_t} \right)^{-\nu}. \tag{A.7}$$

Price stickiness is introduced through a Rotemberg-style price adjustment cost, where retail firm j incurs a cost of $\frac{\psi}{2}\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2$ units of final goods to adjust its nominal price $P_{j,t}$. The optimization problem for retail firm j is to set prices $\{P_{j,s}\}_{s \geq t}$ to maximize the expected discounted sum of real profits that it rebates to the household, discounted by the household's real stochastic discount factor $\Lambda_{t,s}$ for $s \geq t$:

$$\max_{P_{j,s}} \quad \mathbb{E}_t \sum_{s \geq t} \Lambda_{t,s} \left[\frac{P_{j,s}}{P_s} y_{j,s} - m_s y_{j,s} - \frac{\psi}{2} (\frac{P_{j,s}}{P_{j,s-1}} - 1)^2 \right],$$

subject to the demand function for good j given in equation (21). m_t is the real price (in the units of final goods) to purchase goods from bankers' firms. In other words, m_t is the real marginal cost to produce differentiated goods j.

In a symmetric equilibrium, where $P_{j,t} = P_t$ for all j, the optimality condition of the retail firms leads to the New Keynesian Phillips curve.

$$[\nu m_t - (\nu - 1)]\, y_t - \psi \left[(\pi_t - 1)\pi_t - \beta \mathbb{E}_t \Lambda_{t,t+1} \left(\pi_{t+1} - 1 \right) \pi_{t+1} \right] = 0.$$

The government budget constraint is

$$\tau_t w_t h_t + \frac{Q_t^B B_t}{P_t} = \frac{1 + (1-\eta) Q_t^B}{P_t} B_{t-1} + g_t.$$

The social resource constraint takes into account the real adjustment cost that arises from changing prices:

$$c_t + g_t + a_t + \frac{\psi}{2} (\frac{P_t}{P_{t-1}} - 1)^2 = y_t + (1-\delta) a_{t-1}.$$

B Sensitivity analysis

We study the sensitivity of the fiscal financing decomposition results in the baseline economy in Table 3 to various parameters. To conserve space, figures of this section are in the Online Supplement (Figure S.1-S.13).

B.1 Sensitivity to ξ

Figure S.1 presents the sensitivity analysis of the fiscal financing decomposition with respect to the key parameter ξ in the baseline economy with collateral constraints, flexible prices, and short-term government debt. The first three panels of the figure show the contributions of increases in tax revenue, higher current inflation, and changes in the real interest rate, while the last panel presents the debt-to-GDP ratio in the steady state. As the value of ξ increases, the collateral constraint becomes more relaxed, leading to a reduction in the optimal government debt level in the steady state. Consequently, the debt-to-GDP ratio also decreases with higher values of ξ. To enable a meaningful comparison between economies with and without collateral constraints, the steady-state debt-to-GDP ratio in the comparison economy without constraints is adjusted to be the same as in the economy with constraints for each value of ξ.

In the comparison economy without collateral constraints, the contribution of tax revenues is consistently near zero. On the other hand, the contribution of the real interest rate becomes less negative as debt-to-GDP ratio decreases (corresponding to higher values of ξ). This phenomenon occurs because a fiscal shock that increases government consumption leads to a rise in the real interest rate, making debt more costly to service. However, this impact becomes weaker when the debt-to-GDP ratio is smaller. As a result, the contribution of the real interest rate to fiscal financing becomes less negative. This change is accompanied by a declining contribution of inflation.

In the baseline economy with collateral constraints, the contribution of tax revenue increases as ξ rises. This is because a larger ξ results in a smaller debt-to-GDP ratio, a lower labor tax rate and smaller tax distortions in the steady state. When the economy experiences a government consumption shock, the Ramsey policymaker increases the tax rate by more when ξ is larger, as shown in Figure S.2. Accordingly, we observe that the contribution of inflation declines as

ξ increases, ranging from 48% to 65%.[25] Importantly, in the presence of collateral constraints, inflation makes a substantially smaller contribution to fiscal financing compared to the economy without such constraints, regardless of the value of ξ. The contribution of the real interest rate, $\left(-\sum_{s=t+1}^{\infty}\frac{1-(1-\eta)^{s-t}}{(\bar{r}^b)^{s-t+1}}\frac{\bar{b}}{\bar{y}}\hat{r}_s^b\right)/\left(\sum_{s=t}^{\infty}\frac{1}{(\bar{r}^b)^{s-t+1}}\frac{\bar{g}}{\bar{y}}\tilde{g}_s\right)$ (see equation (20)), exhibits a non-monotonic pattern due to two opposing forces. On the one hand, as ξ increases, the high-productivity bankers become less financially constrained in the steady state. Following a government consumption shock and debt monetization, the increase in the liquidity premium of government debt is less pronounced. This results in a smaller decrease (or a larger increase) in the real interest rates, except for the first few periods (see the third panel of Figure S.2). Therefore, $-\sum_{s=t+1}^{\infty}\frac{1-(1-\eta)^{s-t}}{(\bar{r}^b)^{s-t+1}}\hat{r}_s^b$ becomes more negative, as shown in the last panel of Figure S.2. On the other hand, as shown in Figure S.1, the steady-state debt-to-GDP ratio ($\bar{b}/\bar{y}$) diminishes as ξ increases, which tends to moderate the effect of changes in real interest rates. Overall, $-\sum_{s=t+1}^{\infty}\frac{1-(1-\eta)^{s-t}}{(\bar{r}^b)^{s-t+1}}\frac{\bar{b}}{\bar{y}}\hat{r}_s^b$ is non-monotonic in ξ.

B.2 Sensitivity to β

Figure S.3 in the Online Supplement shows the sensitivity of the fiscal financing decomposition concerning the discount factor β, and Figure S.4 presents the impulse response functions in the baseline economy for different values of β to illustrate the mechanisms at play. As β increases, the steady-state real interest rate of government debt in the baseline economy decreases, alleviating the fiscal burden for debt repayment. Therefore, the Ramsey policymaker becomes more inclined to issue debt to relax the collateral constraint on bankers when β is larger, resulting in a higher debt-to-GDP ratio in the steady state. Because of this larger debt-to-GDP ratio, inflation makes a greater contribution to fiscal financing with a larger β, despite the Ramsey planner increasing the inflation rate by a smaller amount following the shock when β is larger (see in Figure S.4). As β increases from 0.96 to 0.998, the contribution of inflation increases from 48% to 60%. Note that in the presence of collateral constraints, inflation makes a substantially smaller contribution to fiscal financing than in the economy without such constraints, regardless of the value of β.

In the steady state of the baseline economy with a larger β, the collateral constraint is less tight due to the larger stock of government debt. Therefore, following the shock, the liquidity premium

[25]Note that in Figure S.2, the response of inflation is larger when ξ is bigger. However, since the debt-to-GDP ratio is smaller, the contribution of inflation is also smaller.

rises by less, leading to a less pronounced decline in the real interest rate in the several periods following the shock (see the last panel of Figure S.4). Consequently, the contribution of real interest rate is more negative with a larger β. In addition, despite the fact that the tax rate increases by a smaller amount in response to the government consumption shock when β is larger, its contribution to fiscal financing is larger. This is mainly because future tax revenues are discounted by a smaller steady-state real interest rate when β is larger.

B.3 Sensitivity to ρ

In this subsection, we explore the sensitivity of the fiscal financing decomposition to changes in ρ. The results are displayed in Figure S.5. In the baseline economy, the contribution of tax revenue increases with larger values of ρ. This is because a smaller intertemporal elasticity of substitution (larger ρ) strengthens the income effect on labor supply (see equation (7)). Consequently, the Ramsey policymaker can raise the labor tax rate without causing a substantial decline in labor supply. Because of the larger contributions by tax revenues, the increase in the inflation rate following the shock is smaller with larger values of ρ, as shown in Figure S.6, leading to a smaller contribution of inflation to fiscal financing. As ρ increases from 0 to 10, the contribution of inflation decreases from 68% to 47%, and it remains substantially lower than that in the comparison economy.

The smaller intertemporal elasticity of substitution (larger ρ) also implies a steeper response in the real interest rate to the shock, as illustrated in the last panel of Figure S.6. The larger increase in the real interest rate (except for the initial few periods) results in a more negative contribution from the real interest rate.

B.4 Sensitivity to δ

This subsection studies the sensitivity of the fiscal financing decomposition to changes in the capital depreciation rate δ, and results are presented in Figure S.7. A larger capital depreciation rate reduces the capital-to-output ratio in the steady state, thus decreasing the amount of government debt needed to facilitate the reallocation of capital. As a result, the debt-to-GDP ratio chosen by the Ramsey policymaker decreases with a larger value of δ.

As shown in Figure S.7, the contribution of tax revenues to fiscal financing increases with δ. This is because, with higher values of δ, the steady-state tax rate is lower, given the reduced debt-to-GDP ratio. Therefore, as shown in Figure S.8, the Ramsey planner has a greater room to raise the tax rate following a government consumption shock. Despite the larger increase in the inflation rate for a larger δ, the contribution of inflation to fiscal financing is smaller due to the lower debt-to-GDP ratio. Specifically, an increase of δ from 0.01 to 0.05 results in a decrease in the contribution of inflation from 72% to 42%. The contribution of inflation in the baseline economy remains significantly smaller than that in the comparison economy for all values of δ we consider. Finally, the contribution of real interest rate somewhat decreases with an increase in δ.

B.5 Sensitivity to σ

We investigate the sensitivity of the results to changes in the probability of a high productivity shock σ, and the findings are displayed in Figure S.9. As σ increases, the fraction of high-productivity bankers increases, and each of them needs to absorb a lower amount of capital from low-productivity bankers in the capital market. Therefore, their need for government debt as collateral declines, leading to a decrease in the optimal debt-to-GDP ratio chosen by the Ramsey planner.

As shown in Figure S.9, the contribution of tax revenues to fiscal financing in the baseline economy increases with σ. This is because, with higher values of σ, the steady-state tax rate is lower, given the reduced debt-to-GDP ratio. Therefore, the Ramsey planner facing a larger σ raises the tax rate by more following a government consumption shock, as it is less distortionary (Figure S.10). In addition, due to the lower debt-to-GDP ratio for a larger value of σ, the contribution of inflation is smaller despite the larger response of inflation following the shock. As σ increases from 0.4 to 0.6, the contribution of inflation to fiscal financing declines from 71% to 44%, and it remains substantially lower than that in the comparison economy. The contribution of real interest rate varies moderately in the baseline economy as the value of σ changes.

B.6 Sensitivity to ϵ

Figure S.11 presents the sensitivity analysis of the fiscal financing decomposition with respect to changes in the inverse Frisch elasticity ϵ. As ϵ varies from 0.2 to 4, representing increasing labor supply inelasticity, the Ramsey policymaker in the baseline economy increases the labor tax rate by more following the government consumption shock, as shown in Figure S.12. This results in an increase in the contribution of taxes from 48% to 64%. Consequently, the response of inflation in the optimal policy is dampened as ϵ increases, leading to a smaller contribution of inflation (from 57% to 47%). For all values of ϵ, the contribution of inflation remains consistently smaller than its counterpart in the comparison economy without collateral constraints.

B.7 Sensitivity to ρ^g

Figure S.13 presents the sensitivity of the fiscal financing decomposition to the persistence of government consumption shock ρ^g. For a discussion of the mechanism, see Section 3.2 of the paper.

C VAR analysis of war financing

We run a VAR model to investigate the impact of war spending on fiscal and monetary policy. The VAR includes the following variables in order: real government consumption expenditure on national defense, CPI inflation, 5-year Treasury yield, real federal government tax receipts, real federal debt, and real GDP. The data, with a quarterly frequency spanning from 2001Q1 to 2023Q3, is used to estimate the VAR model with a lag length of 2.

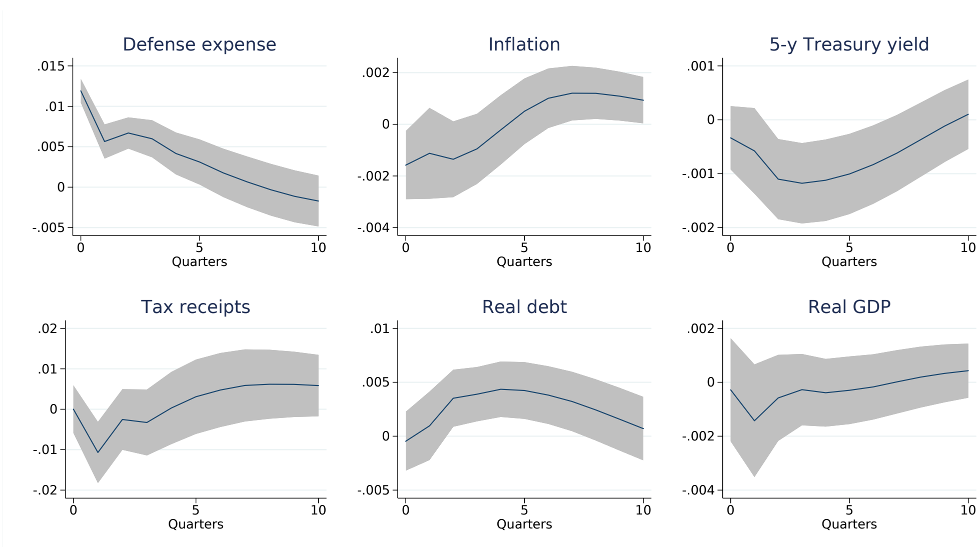

Figure A.1: Impulse response functions to a national defense expenditures shock. The solid line depicts the impulse response function, and the shaded area represents the 90% confidence interval. We apply the natural logarithm to the data on national defense expenditures, real debt, tax revenue, and real GDP, and then apply the HP filter. We apply the HP filter to inflation and bond yield. Data source: Federal Reserve Economic Data.

Figure A.1 displays the impulse response functions to a shock to the national defense expenditures. A positive shock leads to an increase in real debt and a decrease in the Treasury yield. The immediate response of inflation is negative, but it turns positive after 6 quarters.

Online Supplement (Not for Publication)

This supplement contains figures for the sensitivity analysis in Appendix B.

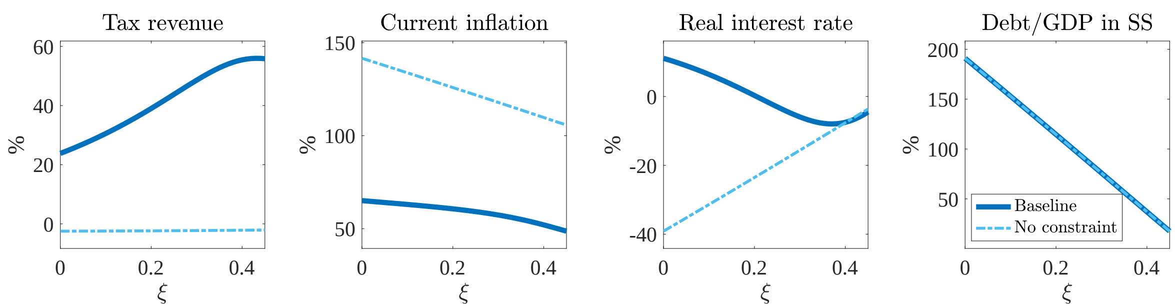

Figure S.1: Sensitivity to ξ. The first three panels show the contributions of higher tax revenue, higher current inflation, and changes in the real interest rate to the fiscal financing, respectively. The last panel presents the debt-to-GDP ratio in the steady state. For each value of ξ, the steady-state debt-to-GDP ratio in the comparison economy without constraints is set to match the corresponding debt-to-GDP ratio in the baseline economy with constraints.

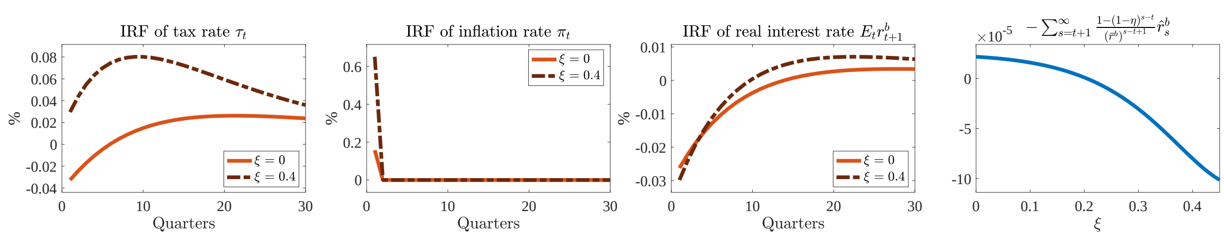

Figure S.2: Understanding the fiscal financing contributions for different ξ in the baseline model. The first three panels display the impulse responses of the tax rate, inflation rate, and real interest rate. The last panel shows the variation of $-\sum_{s=t+1}^{\infty} \frac{1-(1-\eta)^{s-t}}{\left(\bar{r}^b\right)^{s-t+1}} \hat{r}^b_s$ as ξ varies.

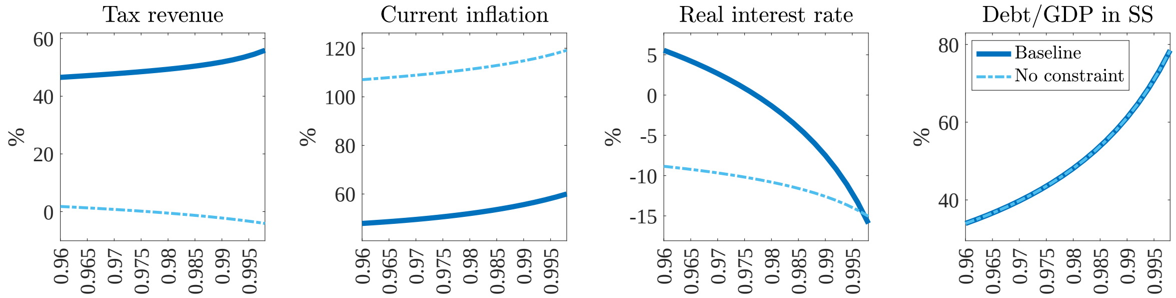

Figure S.3: Sensitivity to β. The first three panels show the contributions of higher tax revenue, higher current inflation, and changes in the real interest rate to the fiscal financing, respectively. The last panel presents the debt-to-GDP ratio in the steady state. For each value of β, the steady-state debt-to-GDP ratio in the comparison economy without constraints is set to match the corresponding debt-to-GDP ratio in the baseline economy with constraints.

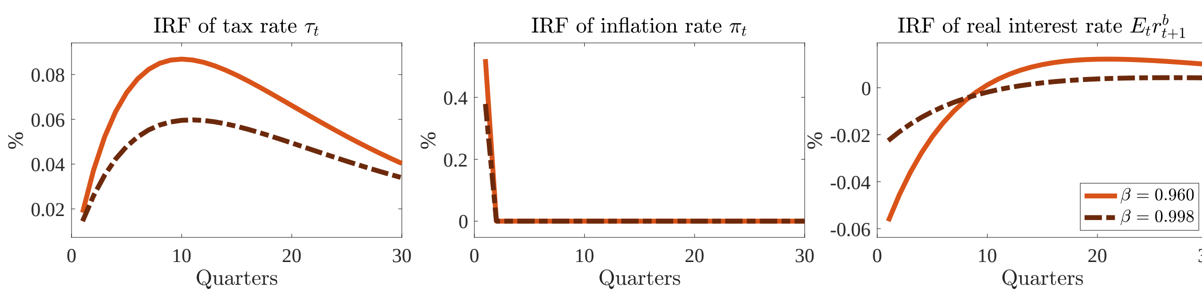

Figure S.4: Impulse response functions in the baseline economy for different values of β

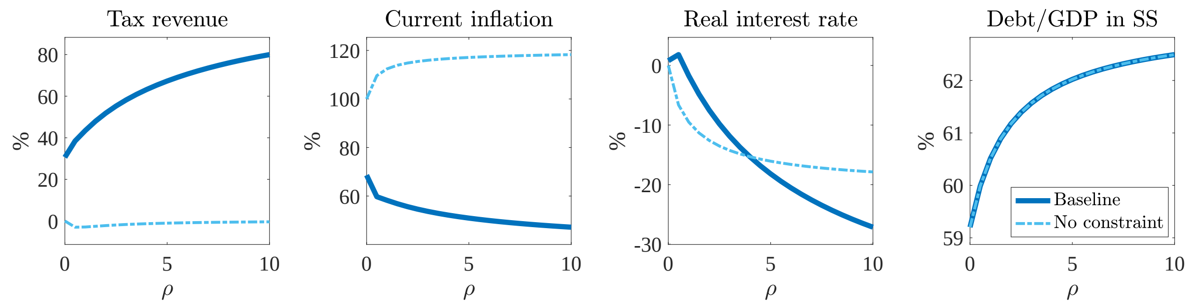

Figure S.5: Sensitivity to ρ. The first three panels show the contributions of higher tax revenue, higher current inflation, and changes in the real interest rate to the fiscal financing, respectively. The last panel presents the debt-to-GDP ratio in the steady state. For each value of ρ, the steady-state debt-to-GDP ratio in the comparison economy without constraints is set to match the corresponding debt-to-GDP ratio in the baseline economy with constraints.

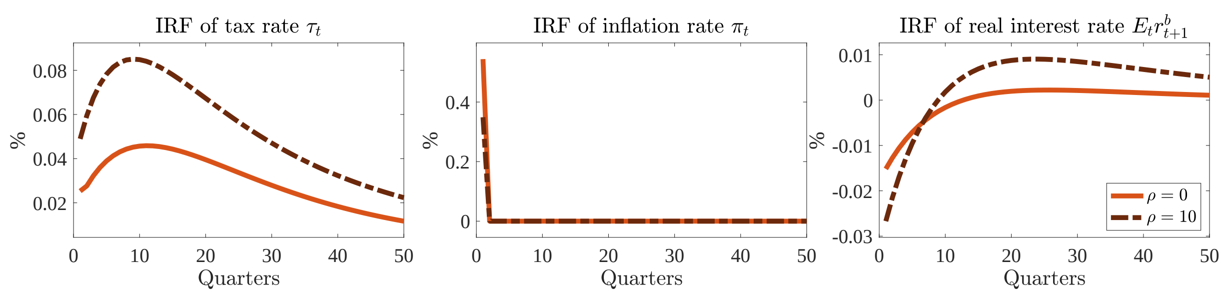

Figure S.6: Impulse response functions in the baseline economy for different values of ρ

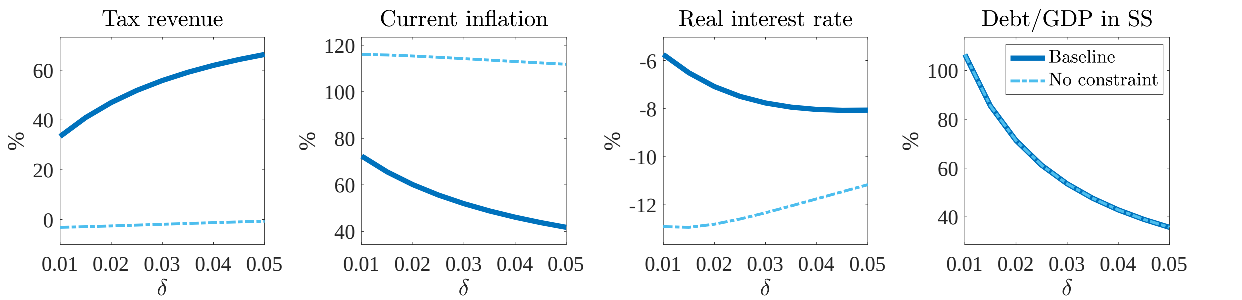

Figure S.7: Sensitivity to δ. The first three panels show the contributions of higher tax revenue, higher current inflation, and changes in the real interest rate to the fiscal financing, respectively. The last panel presents the debt-to-GDP ratio in the steady state. For each value of δ, the steady-state debt-to-GDP ratio in the comparison economy without constraints is set to match the corresponding debt-to-GDP ratio in the baseline economy with constraints.

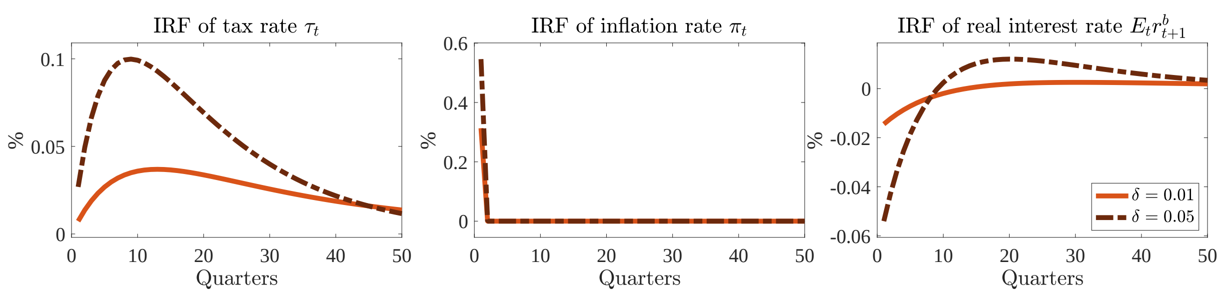

Figure S.8: Impulse response functions in the baseline economy for different values of δ

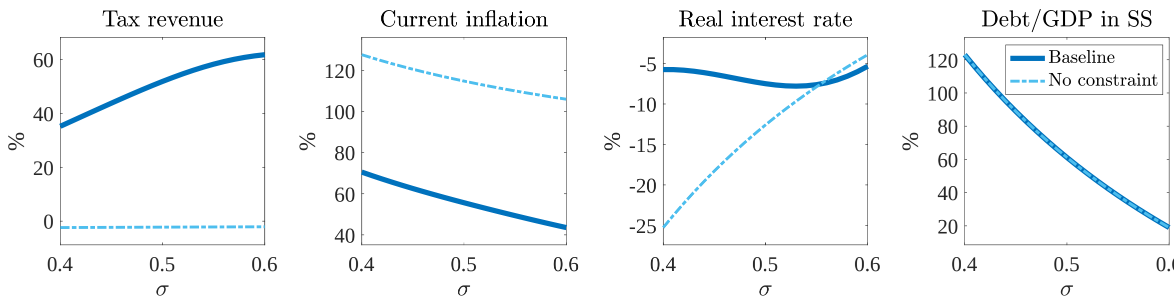

Figure S.9: Sensitivity to σ. The first three panels show the contributions of higher tax revenue, higher current inflation, and changes in the real interest rate to the fiscal financing, respectively. The last panel presents the debt-to-GDP ratio in the steady state. For each value of σ, the steady-state debt-to-GDP ratio in the comparison economy without constraints is set to match the corresponding debt-to-GDP ratio in the baseline economy with constraints.

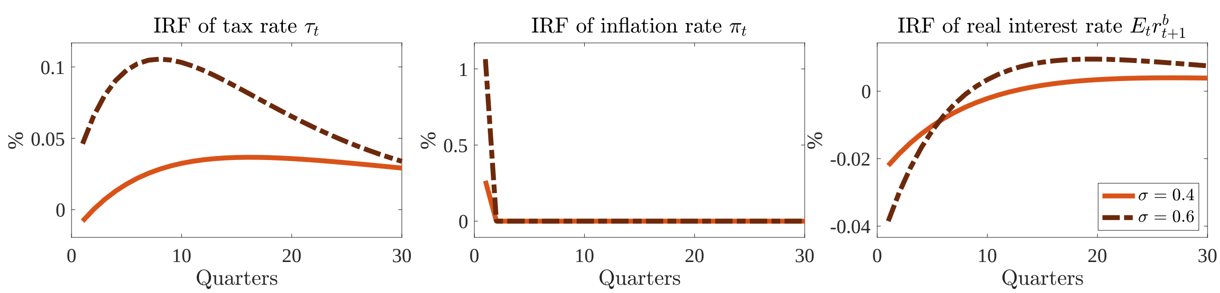

Figure S.10: Impulse response functions in the baseline economy for different values of σ

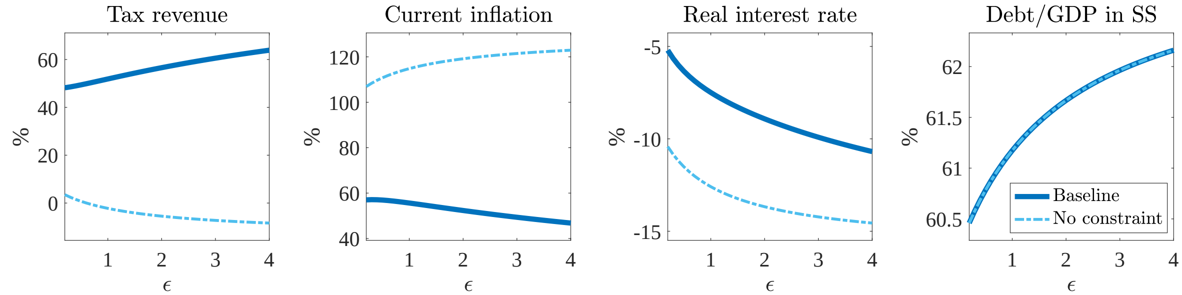

Figure S.11: Sensitivity to ϵ. The first three panels show the contributions of higher tax revenue, higher current inflation, and changes in the real interest rate to the fiscal financing, respectively. The last panel presents the debt-to-GDP ratio in the steady state. For each value of ϵ, the steady-state debt-to-GDP ratio in the comparison economy without constraints is set to match the corresponding debt-to-GDP ratio in the baseline economy with constraints.

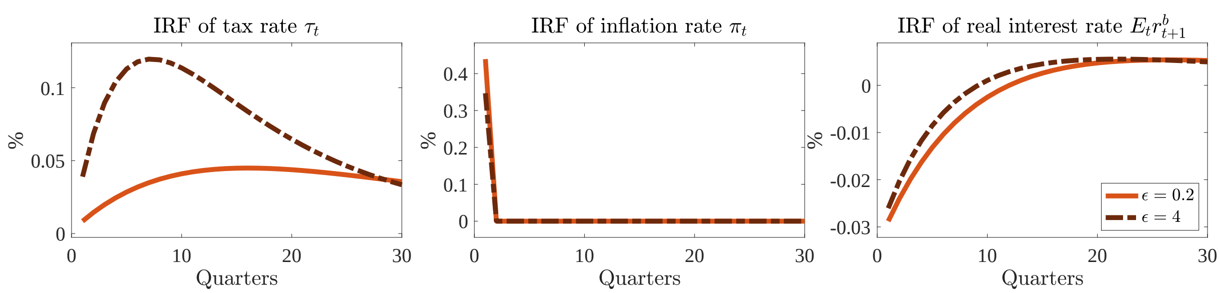

Figure S.12: Impulse response functions in the baseline economy for different values of ϵ

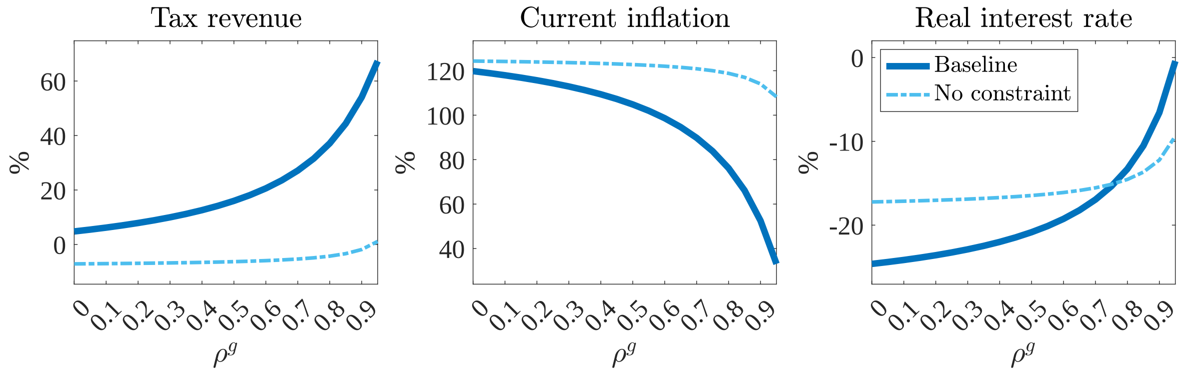

Figure S.13: Sensitivity to ρ^g. The first three panels show the contributions of higher tax revenue, higher current inflation, and changes in the real interest rate to the fiscal financing, respectively. The last panel presents the debt-to-GDP ratio in the steady state. For each value of ρ^g, the steady-state debt-to-GDP ratio in the comparison economy without constraints is set to match the corresponding debt-to-GDP ratio in the baseline economy with constraints.